BIOMETRICS
CSE190
Fall 2006

Assignment 1

Due: October 19, 2006

1. For a single feature, what are the optimal decision regions where:

- $\Omega = \{\omega_1, \omega_2\}$

- $P(x \mid \omega_1) = N(2, 0.5)$ (Normal distribution)
- $P(x \mid \omega_2) = N(1.5, 0.2)$

- $P(\omega_1) = 2/3$
- $P(\omega_2) = 1/3$

- $\lambda_{11} = 0, \lambda_{12} = 1, \lambda_{21} = 1, \lambda_{22} = 0$

2. Select the optimal decision regions for the above problem when:

- $\lambda_{11} = 1, \lambda_{12} = 2, \lambda_{21} = 3, \lambda_{22} = 4$

3.*Problem 2.13 in DHS.* In many pattern classification problems one has the option either to assign the pattern to one of c classes, or to reject it as being unrecognizable. If the cost for rejects is not too high, rejection may be a desirable action. Let

$$\lambda(\alpha_i \mid \omega_j) = \begin{cases} 0 & i = j, i, j = 1, ..., c \\ \lambda_r & i = c + 1 \\ \lambda_s & Otherwise \end{cases}$$

Where λ_r is the loss incurred for chosing the (c+1)th action, rejection, and λ_s is the loss incurred for making any substitution error. Show that the minimum risk is obtained if we decide ω_i if $P(\omega_i \mid \mathbf{x}) >= P(\omega_j \mid \mathbf{x})$ for all j, and if $P(\omega_i \mid \mathbf{x}) = >1-\lambda_r/\lambda_s$, and reject otherwise. What happens if $\lambda_r = 0$? What happens if $\lambda_r > \lambda_s$?

4. *Problem 2.14 in DHS.* Consider the classification problem with rejection option.'
(a) Use the results from problem 2.13 (above) to show that the following discriminant functions are optimal or such problems:

$$g_i(\mathbf{x}) = \begin{cases} P(\mathbf{x} \mid \omega_i) & i = 1,\ldots,c \\ \dfrac{\lambda_s - \lambda_r}{\lambda_s} \sum_{j=1}^{c} P(\mathbf{x} \mid \omega_j) P(\omega_j) & i = c+1 \end{cases}$$

(b) Plot these discrimant functions and decisions regions for the two category, 1-D case having

$P(x \mid \omega_1) = N(1,1)$
$P(x \mid \omega_2) = N(-1,1)$
$P(\omega_1) = P(\omega_2) = 1/2$
$\lambda_r/\lambda_s = 1/4$

(c) Describe qualitatively what happens as λ_r/λ_s is increased from 0 to 1.
(d) Repeat (b) and (c) for the case with

$P(x \mid \omega_1) = N(1,1)$
$P(x \mid \omega_2) = N(0,1/4)$
$P(\omega_1) = 1/3,\ P(\omega_2) = 2/3$
$\lambda_r/\lambda_s = 1/2$

5. Suppose we have two normal distributions with the same convariances but different means: $N(\mu_1, \Sigma)$ and $N(\mu_2, \Sigma)$. In terms of their prior probabilities, $P(\omega_1)$ and $P(\omega_2)$, state the condition that the Bayes decision boundary does **NOT** pass between the two means.

6. Problem 3.2 from DHS. Let *x* have a uniform density

$$p(x \mid \theta) = \begin{cases} 1/\theta & 0 \le x \le \theta \\ 0 & otherwise. \end{cases}$$

Suppose that *n* samples $D = [x_1, \ldots, x_n]$ are drawn independently according to $p(x \mid \theta)$. Show that the maximum likelihood estimate for θ is max[D] – that is, the value of the maximum element in D.