CSE166 – Image Processing – Final
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`http://www-cse.ucsd.edu/classes/fa06/cse166`
11:30am-2:30pm Tue. Dec. 5, 2006.

On this exam you are allowed to use a calculator and two 8.5" by 11" sheets of notes. The total number of points possible is 58. Good luck!

Part I: Fill in the Blank (1 pt. each, 20 pts. total).

1. The Fourier transform of a Gaussian is a(n) ______________ .
2. If a function is narrow in the spatial domain, then it is ______________ in the frequency domain.
3. $\nabla I(x, y)$ is computed in Matlab using the function ______________ .
4. Given a function $f(x)$, the M-point DFT $F(u)$ evaluated at $u = (M/2) + 1$ is also known as the ______________ -component.
5. The binomial function is the discrete approximation to a ______________ function.
6. A neighborhood of an image where half the gradient vectors equal $(\pm 1, 0)$ and the other half equal $(0, \pm 1)$ is an example of a rank-______________ neighborhood.
7. The values u and v satisfying the equation $I_x u + I_y v + I_t = 0$ in a small window represent the ______________ .
8. The convolution of an $M \times M$ image with an $N \times N$ kernel is of size ______________ .
9. The image enhancement operation that makes the probability density function of pixel brightnesses approximately uniform is called ______________ .
10. ______________ is an example of a lossless image compression method.
11. The lower bound in lossless image compression is set by the ______________ of the source.
12. When computing the ______________ transform, each pixel in the (x, y) domain produces a sinusoid in the (ρ, θ) domain.
13. We compute $I(x, y, t) - I(x, y, t - 1)$ as an approximation of ______________ .
14. The ambiguity of motion viewed in a small window is known as the ______________ problem.
15. True or False: JPEG is recommended for compressing images of natural scenes. ______________
16. An image with an approximately flat histogram has ______________ entropy than an image with a highly peaked histogram .
17. Given an RGB image, the ______________ channel representation is given by the three images $R - G$, $B - (R + G)/2$ and $(R + G + B)/3$.
18. The morphological operation that fattens pixels around object boundaries is called ______________ .
19. In the HSI color space, the length of the vector extending from the axis of the cone represents the ______________ and the angle represents the ______________ .
20. If you fixate on an image of an American flag for 60 seconds and subsequently view a blank white screen, you will see the following three colors in place of red, white and blue: ______________ , ______________ , and ______________ , respectively.

Part II: Written problems.

1. (6 pts.) Write down the steps of k-means clustering, including the initialization, the basic iteration, and the stopping criterion. Illustrate your answer with a 2D pointset example with two clumps. Set $k = 2$ and show iterations 0, 1 and 2. Assume the initialized cluster centers both fall within one of the clumps.

2. (13 pts) This problem makes use of the binary image displayed in Figure 1, in which black=1 and white=0. Note: in calculating the various quantities in this problem, round your answers to 2 significant figures.

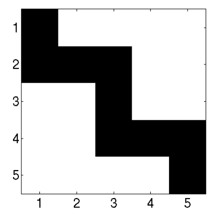

Figure 1: 5×5 binary image.

   (a) Compute the coordinates of the centroid $\mathbf{m}$.
   (b) Compute the scatter matrix C.
   (c) Find the eigenvalues λ_1 and λ_2 of C and use them to compute the aspect ratio.
   (d) Find the angle ϕ of the principal eigenvector of C. Also write down the angle of the 2nd eigenvector. Express each answer in units of degrees.
   (e) Letting $\mathbf{x}_k$ denote the original coordinates of the nonzero pixels, find the values of the rotation matrix R and translation vector $\mathbf{t}$ in the expression

$$\mathbf{x}'_k = R(\mathbf{x}_k + \mathbf{t}), \quad k = 1, 2, \ldots, 9$$

   such that the set of transformed coordinates $\mathbf{x}'_k$ for $k = 1, 2, \ldots, 9$ is centered at the origin and has its principal axis aligned with the y axis.

3. (5 pts.) Define $h(x)$ to be the centered first difference kernel. Now define the function

$$H(u) = \sum_{x=-\infty}^{\infty} h(x) e^{-j2\pi ux}$$

   In this problem, x is discrete and u is continuous.

   (a) What does $H(u)$ represent?
   (b) Plug in the values of $h(x)$ to solve for $H(u)$, and write it in its simplest form.
   (c) Sketch $H(u)$.
   (d) Based on the shape of $H(u)$, what type of filter is $h(x)$?

4. (4 pts.) Prove that the eigenvalues of a covariance matrix are non-negative. What is this property called?