Homework #9: IS Review, Equivalence, & Pigeonhole Principle
Due Date: December 1, 2005
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Solutions to IS Review $\{10, 11\}$ and EO $\{1.6, 1.14, 1.15, 1.22\}$ are in the book.

Comments:

- IS Review 11 - Why does the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n n^{500}}{(1.0001)^n}$$

converge absolutely? Check if the summation using the sequence $|a_n|$ converges. Taking the absolute value is equivalent to ignoring the term $(-1)^n$, since the resulting sequence will be positive.

If $|a_n|$ converges faster than a sequence b_n where $\sum b_n$ converges absolutely, then $\sum |a_n|$ converges absolutely.

In the chapter it is shown that $\sum \frac{1}{n^2}$ converges, so use $\frac{1}{n^2}$ as b_n. Compare values when $n \to \infty$ with a limit expression

$$\lim_{n\to\infty} \frac{\frac{n^{500}}{(1.0001)^n}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^{502}}{(1.0001)^n}$$
$$= 0$$

We know the limit is zero by applying L'Hopital's Rule ≈ 503 times. Each application reduces the degree of the polynomial in the numerator by 1, but the denominator stays approx exponential. Since the limit is 0, the expression originally in the numerator ($|a_n|$) has smaller values as $n \to \infty$ and converges faster.

Website Problems

Group of 30 people

In a group of 30 people, must at least 3 have been born in the same month? Why?

Solution: The avg number of people born per month is $30/12 = 2.5$. Since the max $\geq avg$ and the number of people born per month must be integer, max ≥ 3. The max occurs some month, so that month is an example.

A different approach is to consider, what is the minimum number of months before it must be true that at least 3 are born in some month. This is minimum is $12 \cdot (3-1)+1 = 25$ months. Since $30 \geq 25$, it must be true for 30 months as well.

Group of 30 people II

In a group of 30 people, must at least 4 have been born in the same month? Why?

Solution: Just find a counter-example: 3 people are born every month from January to October and 0 people after that.

Hard Coding

A programmer writes 500 lines of computer code in 17 days. Must there have been at least one day when the programmer wrote 30 or more lines of code. Why?

Solution: Apply the same logic as the previous problems. The average is $500/17 \approx 29.4$. Since there are no fractional lines, the max must be ≥ 30.

Subset Sum

Imagine a function f that maps subsets of 4 integers to its sum. For example, $f(\{8,24,34,35\}) = 101$ and $f(\{9,18,24,50\}) = 101$. What is the size of the Domain and Image/Coimage?

The Domain is all possible subsets of 4 integers chosen from a pool of 10. Therefore $|Domain(f)| = \binom{10}{4} = 210$.

The Image involves sums of integers chosen from $1,\ldots,50$. The smallest sum is $1+2+3+4 = 10$. The largest sum is $50+49+48+47 = 194$. The Image can take on values between those limits, so the $|Image(f)| \leq 194-10+1 = 185$ ($\leq$ because it might be impossible to take on all values in the range with only 10 numbers). Since the $|Image(f)| < |Domain(f)|$, the function f cannot be one-to-one. Thinking back about the meaning of the function, this means at least two different subsets have the same sum.