Homework #6: Sets and Functions
Due Date: November 3, 2005
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Solutions to SF {1.15, 1.16, 1.19, 1.20, 2.4, 2.6, 2.9, 2.12} are in the book.

Prove using the element method that, for all subsets P, Q, R of U, $Q - (P \cap R) = Q - (P \cap Q \cap R)$.

Pick $x \in Q - (P \cap R)$.

$$\begin{aligned}
Q - (P \cap R) &= Q \cap (P \cap R)^c \\
&= Q \cap (P^c \cup R^c) \\
&\text{so } x \in Q \text{ and} \\
&\quad (x \notin P \text{ or } x \notin R)
\end{aligned}$$

Recall that $B \vee false = B$ for any boolean expression B. Note, that if $x \in Q = true$ then $x \notin Q = false$.

$$\begin{aligned}
x \notin P \vee x \notin R &= x \notin P \vee x \notin R \vee x \notin Q \\
&\text{so } x \notin (P \cap Q \cap R) \text{ and we already know} \\
&\quad x \in Q \\
&\text{therefore } x \in Q - (P \cap Q \cap R)
\end{aligned}$$

So far, this shows that $Q-(P\cap R) \subseteq Q-(P\cap Q\cap R)$. To show $Q-(P\cap R) \supseteq Q-(P\cap Q\cap R)$, pick $y \in Q - (P \cap Q \cap R)$.

$$\begin{aligned}
Q - (P \cap Q \cap R) &= Q \cap (P^c \cup Q^c \cup R^c) \\
&\text{so } y \in Q \wedge (y \notin P \vee y \notin Q \vee y \notin R) \\
&\quad = y \in Q \wedge (y \notin Q \vee (y \notin P \vee y \notin R)) \\
&\quad = (y \in Q \wedge y \notin Q) \vee (y \in Q \wedge (y \notin P \vee y \notin R)) \\
&\quad = false \vee (y \in Q \wedge (y \notin P \vee y \notin R)) \\
&\quad = y \in Q \wedge (y \notin P \vee y \notin R) \\
&\text{so } y \in (Q \cap (P^c \cup R^c)) \\
&\quad y \in (Q - (P \cap R))
\end{aligned}$$

This shows $Q-(P\cap R) \supseteq Q-(P\cap Q\cap R)$. Along with the earlier result that $Q-(P\cap R) \subseteq Q - (P \cap Q \cap R)$, we've shown $Q - (P \cap R) = Q - (P \cap Q \cap R)$.