CSE 20: Week 1 Howework Solutions

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September 30, 2005

BF: Solutions to 1.3, 1.5, 1.15, 2.3, 2.6 are found in the back of your textbook.

BF 1.13: Is the function $\sim (p \vee \sim q) \vee (\sim p \wedge \sim q)$ equal to the function $\sim p$? Why or why not?

Our proof will use the follow truth table:

p	q	$\sim (p \vee \sim q)$	$(\sim p \wedge \sim q)$	$\sim (p \vee \sim q) \vee (\sim p \wedge \sim q)$	$\sim p$
0	0	0	1	1	1
0	1	1	0	1	1
1	0	0	0	0	0
1	1	0	0	0	0

Since the two function have the same output value for each of the input assignments, the functions are equivalent.

BF 2.7: The solution in the textbook is incorrect. To see this, write out the truth table. The correct answer is $p \wedge (\sim q \vee r)$.

1: Modify the half-adder circuit to use only AND, OR, and NOT gates.

The carry bit c remains $(p \wedge q)$ while the sum bit s can be written as $(\sim p \wedge q) \vee (p \wedge \sim q)$.

2: Prove that the square of an odd integer is an odd integer.

Def: n is an odd integer if and only if there exists an integer k such that $n = 2(k) + 1$.

Proof:

- Let m be an odd integers.
- Then there exists an integer j such that $m = 2(j) + 1$.
- The square of the odd integer m can be written as

$$\begin{aligned} m^2 &= (2j+1)^2 && (1) \\ &= 4j^2 + 4j + 1) && (2) \\ &= 2(2j^2 + 2j) + 1 && (3) \end{aligned}$$

- Since the product of two integers is an integer, $(2j^2)$ and $2j$ are integers.
- Since the sum of two integers is an integer, $2j^2 + 2j$ is an integer.
- m^2 is an odd integer since there exist an integer $(2j^2 + 2j)$ such that $2(2j^2 + 2j) + 1 = m^2$
- Therefore, the square of an odd integer is odd.

3: Prove that the difference of two odd integers is an even integer.

Def: n is an odd integer if and only if there exists an integer k such that $n = 2(k) + 1$.

Def: n is an even integer if and only if there exists an integer k such that $n = 2(k)$.

Proof:

- Let m and n be two odd integers.
- Then there exists two integers j and k such that m = 2(j)+1 and n = 2(k)+1.
- The difference between two odd integers is

$$\begin{aligned} m - n &= (2j + 1) - (2k + 1) && (4) \\ &= 2(j - k) && (5) \end{aligned}$$

- Since the difference between two integers is also an integer, $(j - k)$ is an integer.
- $m - n$ is an even integer since there exist an integer $(j - k)$ such that $2(j - k) = m - n$
- Therefore the difference of two odd integers is an even integer.