

CSE 20—Discrete Math

Fall, 2005

Day 9
Sets

Instructor: Neil Rhodes

Definitions

Set: Contains zero or more *elements*, repetitions ignored, order doesn't matter

- Example with no elements
 - $\{\}$ (alternative syntax: $\emptyset$)
- Example with three elements
 - $\{\text{john, sally, harry}\} = \{\text{harry, sally, john}\} = \{\text{harry sally, harry, john}\}$
- Example with infinite elements
 - $\{x: x \text{ is a positive integer}\}$
- What about:
 - $\{\emptyset\}$

Ordered set: Contains zero or more elements, repetitions *not* allowed, order *does* matter (not often used)

- $(\text{john, sally, harry}) \neq (\text{sally, harry, john})$

Ordered list: Contains zero or more elements, repetitions allowed, order *does* matter (used often)

- Ordered 4-tuple
 - $(\text{john, sally, harry, sally})$
- Ordered pair
 - (x, y)
- Ordered triple
 - (x, y, z)

2

Set notation

$x \in S$

- x is an element of S

$S \subseteq T$

- S is a subset of T : $\forall x, x \in S \rightarrow x \in T$

$S \subset T$

- S is a proper subset of T : $S \subseteq T \wedge (\exists x: x \in T \wedge x \notin S)$

$S = T$

- S is equal to T : $S \subseteq T$ and $T \subseteq S$

$|S|$

- The cardinality (size) of set S : the number of elements of S

k -set

- A set with cardinality k

$\{x: x > 1\}$ (or $\{x \mid x > 1\}$)

- Defines the set by stating the properties shared by the elements

3

Set Operations

$S \cup T$

- Union: $\{x: x \in S \vee x \in T\}$

$S \cap T$

- Intersection: $\{x: x \in S \wedge x \in T\}$

$S - T$ (or $S \setminus T$)

- Difference: $\{x: x \in S \wedge x \notin T\}$

Often we have some universe U , in mind

S^c (or $\sim S$)

- Complement of S : $U - S$

$S \oplus T$

- Symmetric difference: $(S - T) \cup (T - S)$

$S \times T$

- (Cartesian) product: $\{(x, y): x \in S \wedge y \in T\}$

4

Venn Diagrams

Rectangle is universe
Circle (or ellipse) for each set
Overlap represents intersection

Example:

- $A \subseteq B$
- $A \cap B$
- $A \cap B^c$

5

Proof Using Venn Diagrams

Warning: has limitations

Proving $LHS=RHS$ (or $LHS \neq RHS$)

- Create general Venn diagram for given sets
- (optional) Number each region
 - (Given k sets, how many regions?)
- Shade in regions for LHS
- Shade in regions for RHS
- Compare

6

Example Proof

Prove $(A \cap B)^c = A^c \cup B^c$

7

Proof Using Elements

To prove $LHS=RHS$

- Prove $LHS \subseteq RHS$
- Prove $RHS \subseteq LHS$

To prove $LHS \subseteq RHS$

- If $x \in LHS$, then ...
- Thus, $x \in RHS$
- This shows that $LHS \subseteq RHS$

Example: prove $(A \cap B)^c = A^c \cup B^c$

- If $x \in (A \cap B)^c$, then $x \notin (A \cap B)$
- Thus, $x \notin A$ or $x \notin B$.
- So, $x \in A^c$ or $x \in B^c$
- Hence, $x \in A^c \cup B^c$
- This shows that $(A \cap B)^c = A^c \cup B^c$

8

Algebraic Rules for Sets

Associative	$A \cup (B \cap C) = (A \cup B) \cap C$	$A \cap (B \cup C) = (A \cap B) \cup C$
Distributive	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
DeMorgan's	$(A \cup B)^c = A^c \cap B^c$	$(A \cap B)^c = A^c \cup B^c$
Idempotent	$A \cup A = A$	$A \cap A = A$
Double Negation	$(A^c)^c = A$	
Absorption	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$
Commutative	$A \cup B = B \cup A$	$A \cap B = B \cap A$

9

Proof Using Algebraic Rules

$$A = (B^c \cup A) \cap (A \cup B)$$

10

Tabular Method

$$A = (B^c \cup A) \cap (A \cup B)$$

Very similar to truth tables

A	B	C	B^c	$B^c \cup A$	$A \cup B$	$(B^c \cup A) \cap (A \cup B)$
€	€	€				
€	€	€				
€	€	€				
€	€	€				
€	€	€				
€	€	€				
€	€	€				
€	€	€				

11

Tabular Method vs. Venn Diagram

12