

CSE 20—Discrete Math

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Boolean Functions

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Miscellaneous

See website:

www.cse.ucsd.edu/classes/fa05/cse20

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Overview

This course provides the mathematical background necessary for reasoning in computer science. Students will learn logic and basic proof techniques useful for the design and verification of programs and circuits.

Topics:

- Boolean functions and Computer Arithmetic
- Logic
- Number Theory/Cryptography
- Sets/Functions
- Induction/Sequences/Series
- Equivalence/Order

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Boolean Functions

Give two sets A and B , a function is a rule that tells how to find a unique $b \in B$ for each $a \in A$.

- It is shown as $f: A \rightarrow B$
- A is the *domain*
- B is the *range* (or *codomain*)

Boolean functions map from the cartesian product $\times^n \{0, 1\}$ to $\{0, 1\}$

- They take n boolean inputs and produce one boolean output
- How many boolean functions are there that take 0 inputs?

- 1 input?

- 2 inputs?

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Truth Tables

Contain one row for each combination of possible inputs, along with output

- p
- NOT p
- p AND q ($p \wedge q$)
- p OR q ($p \vee q$)
- p XOR q ($p \oplus q$)

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Constructing Boolean Functions

Operators:

- Unary:
 - $\sim$
- Binary (common):
 - $\wedge$ (AND), $\vee$ (OR), $\oplus$ (XOR)
- Precedence: Unary not has higher precedence than binary operators

Equivalence: Two boolean functions are equal if and only if they have the same truth table

- Example: $\sim((p \wedge q) \vee (\sim p \wedge \sim q)) =$

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Truth Table $\rightarrow$ Boolean Function

Disjunctive Normal Form

p	q	r	z
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

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How Computers Add

We want to add two binary numbers:

13
+22

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Rules:

- $0 + 0 =$
- $0 + 1 =$
- $1 + 0 =$
- $1 + 1 =$

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Half Adder (Single-bit Adder)

Truth Table:

p	q	s	c
0	0		
0	1		
1	0		
1	1		

Black box:

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Half Adder

Calculating the carry bit

p	q	s	c
0	0		
0	1		
1	0		
1	1		

Circuit:

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Half Adder

Calculating the sum bit

A	B	S	C
0	0		
0	1		
1	0		
1	1		

Circuit:

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Half Adder

A	B	S	C
0	0		
0	1		
1	0		
1	1		

Overall circuit:

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Other Gates

Or

Invert (not)

Nand

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Combinatorial (Combinational) Circuit

An input wire can split to be used as input to two gates

An output wire can be used as input

The outputs depend only on the present inputs

- Don't depend on history
- One way to achieve this: no cycles
- Sequential circuits: output depends on the history

Direct correspondence between combinatorial circuits and boolean functions

- Master's thesis by Claude Shannon: 1937

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Full Adder

Half-adder only works for rightmost bits of two numbers.

Full adder can also handle the incoming carry bit

Circuit?

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Decoder

Looking for special input, signaled with a 1 on an output line

- Specific decoder looking for 011

General 3-to-8 binary decoder. The n th of 8 output lines is 1 if and only if the 3 incoming lines represent n in binary

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Multiplexer

3-8 Multiplexer. Output line has value of one of 8 input lines as specified by 3 address lines
