CSE166 – Image Processing – Midterm
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`http://www-cse.ucsd.edu/classes/fa05/cse166`
11:00am-12:20pm Tuesday Nov. 1, 2005.

On this exam you are allowed to use a calculator and one 8.5" by 11" sheet of notes. The total number of points possible is 30. In order to get full credit you must **show all your work**. Good luck!

1. (4 pts) Let the vector $(R, G, B)^\top$ denote the RGB color values of a pixel. Consider the following linear transformation:
$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 1 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

   (a) What is the color representation given by $(C_1, C_2, C_3)^\top$ known as?

   (b) Name one practical advantage of using $(C_1, C_2, C_3)^\top$ instead of $(R, G, B)^\top$.

   (c) Can the transformation from RGB to HSI also be written as a linear transformation? If so, give the corresponding 3×3 matrix. If not, explain why.

2. (8 pts) Consider the $M \times N$ image $f(x, y) = (-1)^{x+y}$ for $x = 0, \dots, N-1$ and $y = 0, \dots, M-1$.

   (a) Describe what $f(x, y)$ looks like as an image.

   (b) Put $f(x, y)$ into the form $e^{j2\pi \mathbf{u}_o \cdot \mathbf{x}}$, where $\mathbf{x} = (x, y)^\top$ and $\mathbf{u}_o = (u_o, v_o)^\top$.

   (c) What is the Fourier transform $F(u, v)$? Write it down for the case of $M = N = 4$.

   (d) Suppose you pointwise-multiply an image by $f(x, y)$ and then compute the Fourier transform of the product. What is the motivation for doing this? What Matlab function can be used for this purpose?

3. (8 pts) Let $r_k, k = 0, \dots, L-1$, denote the gray levels for an image $f(x, y)$ with normalized histogram (or pdf) $p(r_k)$.

   (a) Give the expression for the entropy of the image, denoted H.

   (b) Sketch what $p(r_k)$ looks like for a typical natural image.

   (c) Suppose you convolve $f(x, y)$ with an LoG kernel to obtain the filtered image $g(x, y)$. Sketch what the pdf of $g(x, y)$ will look like. What is the name of this type of pdf?

   (d) Is the entropy of $g(x, y)$ greater or less than $f(x, y)$? Which image is more compressible?

4. (10 pts) Consider the 1D continuous kernel $h(x) = e^{-x^2/2\sigma^2} e^{-j2\pi u_o x}$.

   (a) Give the name of this kernel.

   (b) What type of filter is $h(x)$: lowpass, bandpass, or highpass?

   (c) Sketch an example of $h(x)$ and its Fourier transform $H(u)$. (Draw the real and imaginary parts separately.) Label your drawings to show what σ and u_o control.