Interactive Graph Cuts

for Optimal Boundary & Region Segmentation of Objects in N-D Images

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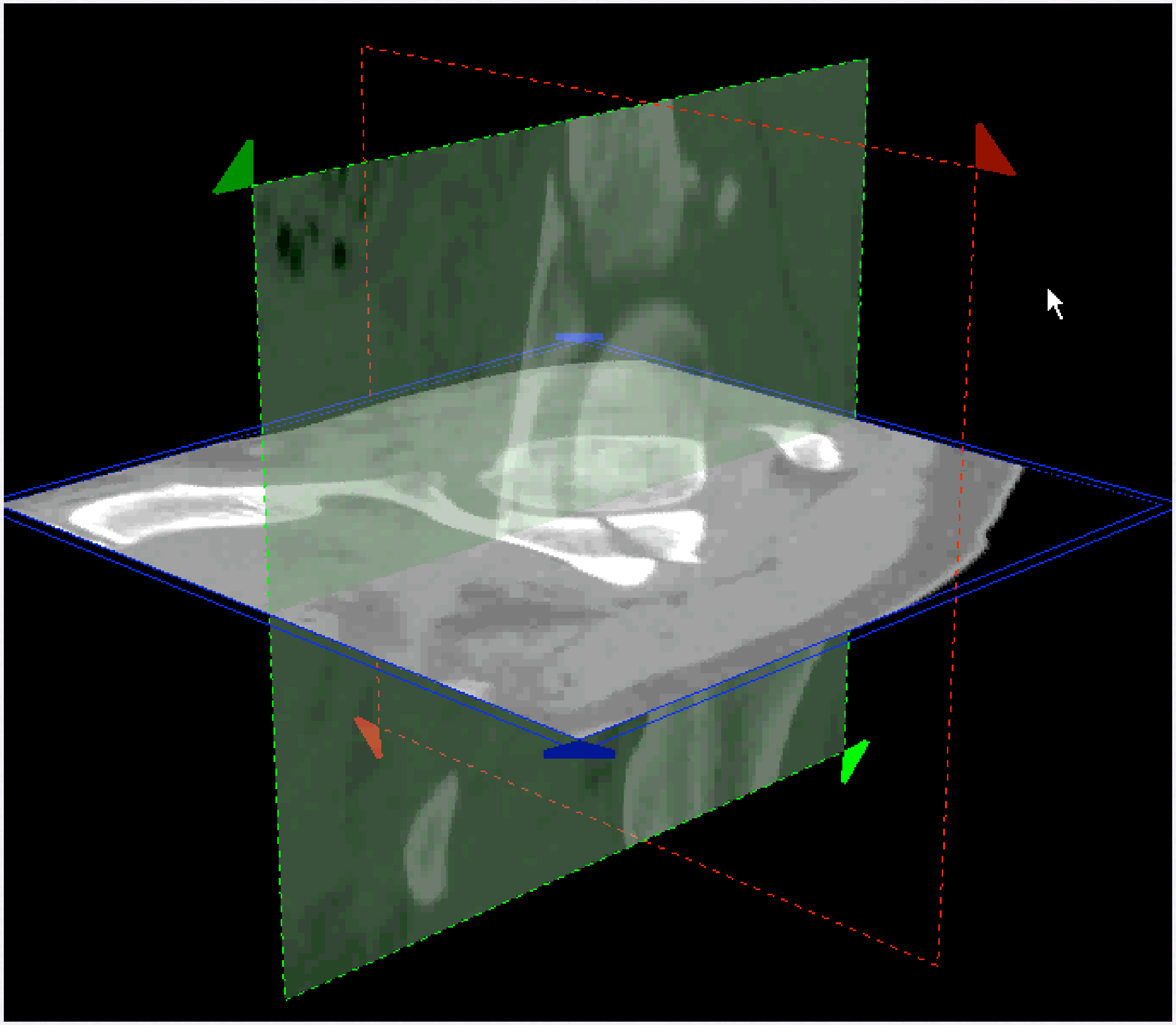
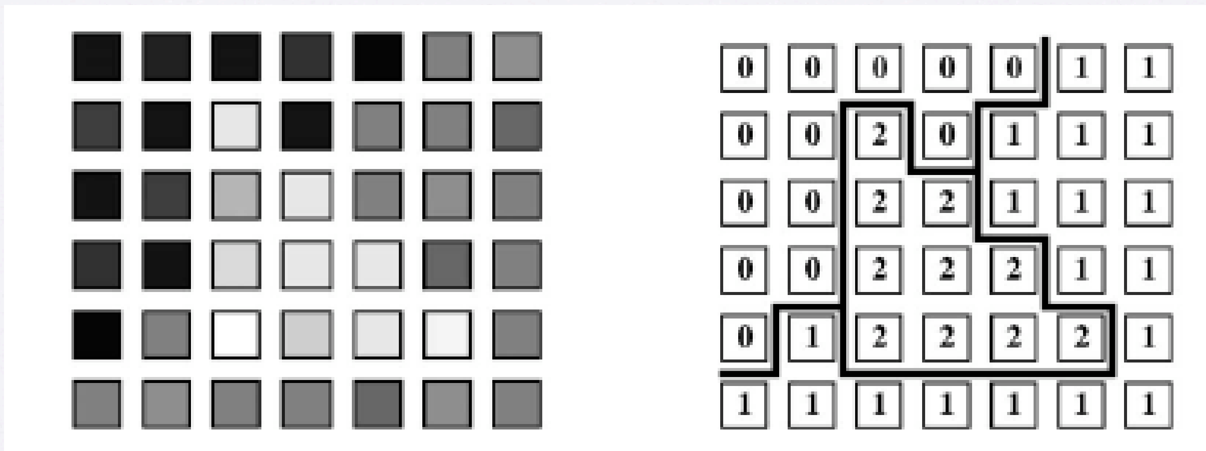


From [3]

Vision problems as image labeling



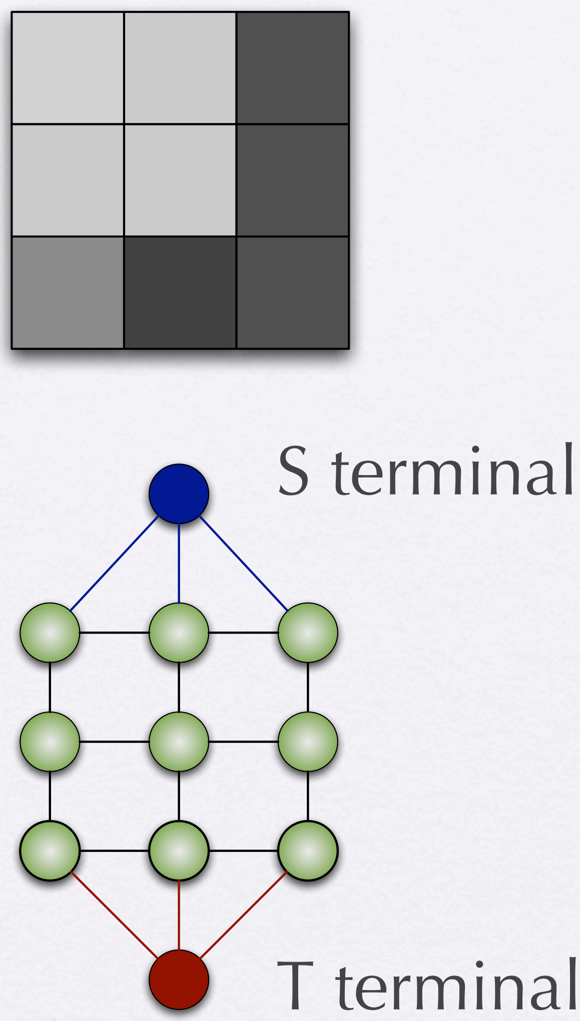
- depth (stereo)
- object index (segmention)
- original intensity (image restoration)

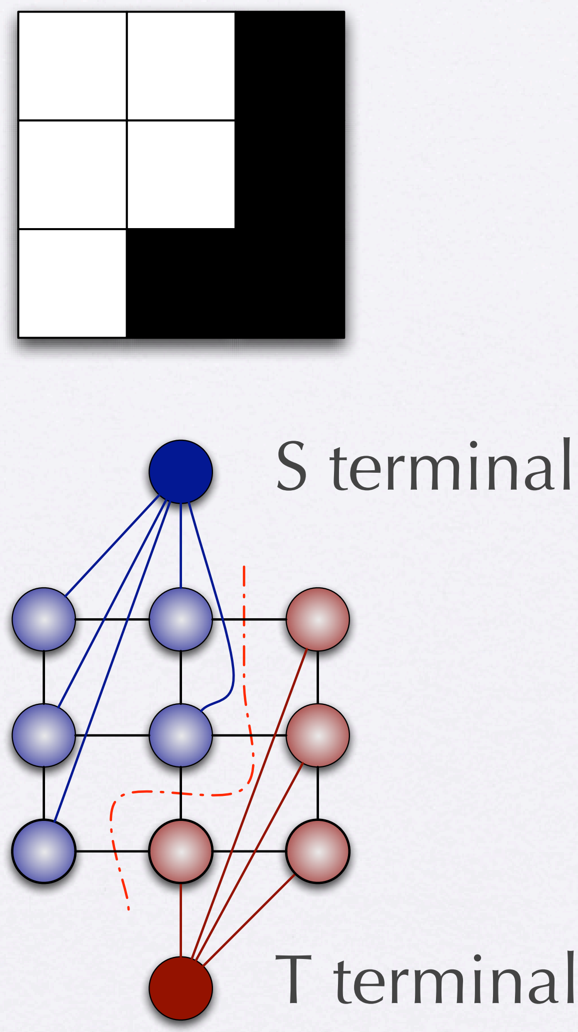
Labeling problems can be cast in terms of energy minimization

$$E(L) = \sum_{p \in P} D_p(L_p) + \sum_{p,q \in N} V_{p,q}(L_p, L_q)$$

$L : P \rightarrow L_p$	Labeling of pixels
$D_p : L_p \rightarrow \Re$	Penalty for pixel labeling
$V_{p,q} : P \text{ x } P \rightarrow \Re$	Interaction between neighboring pixels. Smoothing term.

Energy minimization can be solved with graph cuts





- Energy function and graph construction
- Min-cut of graph minimizes energy
- Summar max-flow/min-cut algorithms
- Rest of bone segmentation example

Some Notation

$P =$ set of pixels p

$N =$ set of unordered pairs (p, q) of neighbors in P

$L = (L_1, \ldots, L_p, \ldots, L_{|P|})$ Binary vector representing a binary segmentation

$G = (V, E)$ graph with nodes, V, and edges, E

$\mathcal{B} =$ set of user defined background pixels

$\mathcal{O} =$ set of user defined object pixels

Labeling problems can be cast in terms of energy minimization

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Their Energy Function

$$E(L) = \lambda \cdot \sum_{p \in P} R_p(L_p) + \sum_{(p,q) \in N} B(p,q) \cdot \delta(L_p, L_q)$$

$D_p(L_p)$ becomes regional term $R_p(L_p)$

$V_{p,q}$ becomes boundary term $B(p,q) \cdot \delta(L_p, L_q)$

Regional term

$$E(L) = \lambda \cdot \sum_{p \in P} R_p(L_p) + \sum_{(p,q) \in N} B(p,q) \cdot \delta(L_p, L_q)$$

Penalize pixel label based on local properties

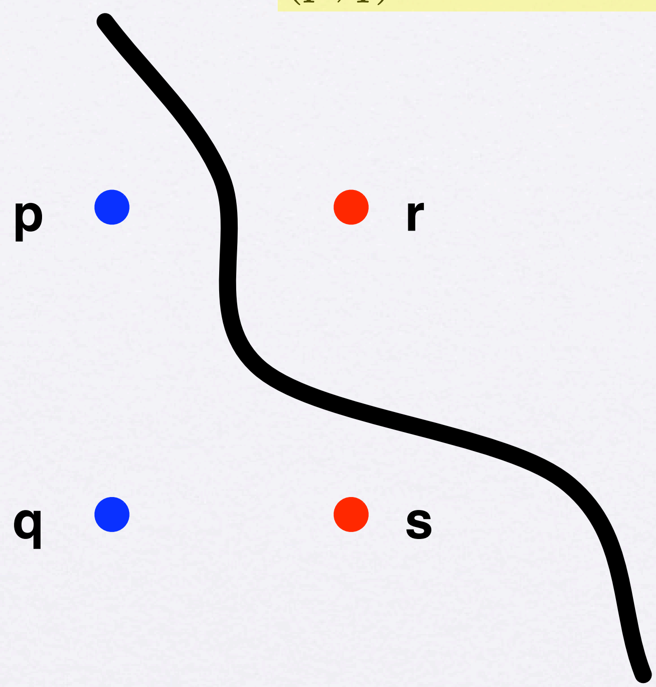
Negative log-likelihood of intensity

$$R_p(obj) = -\ln Pr(I_p|\mathcal{O})$$

$$R_p(bkq) = -\ln Pr(I_p|\mathcal{B})$$

Boundary term: penalize dissimilar neighbors

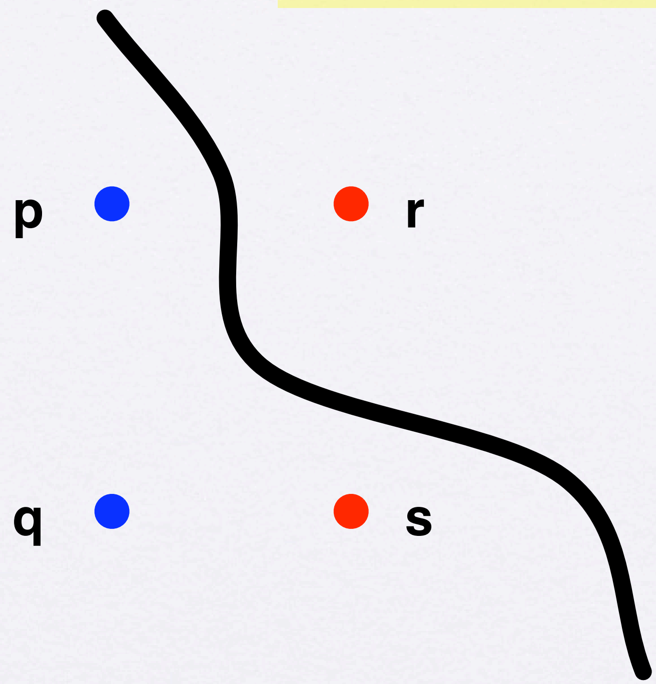
$$E(L) = \lambda \cdot \sum_{p \in P} R_p(L_p) + \sum_{(p,q) \in N} B(p,q) \cdot \delta(L_p, L_q)$$



$$B(p,q) \propto \exp\left(-\frac{(I_p - I_q)^2}{2\sigma^2}\right) \cdot \frac{1}{dist(p,q)}$$

Boundary term: penalize dissimilar neighbors

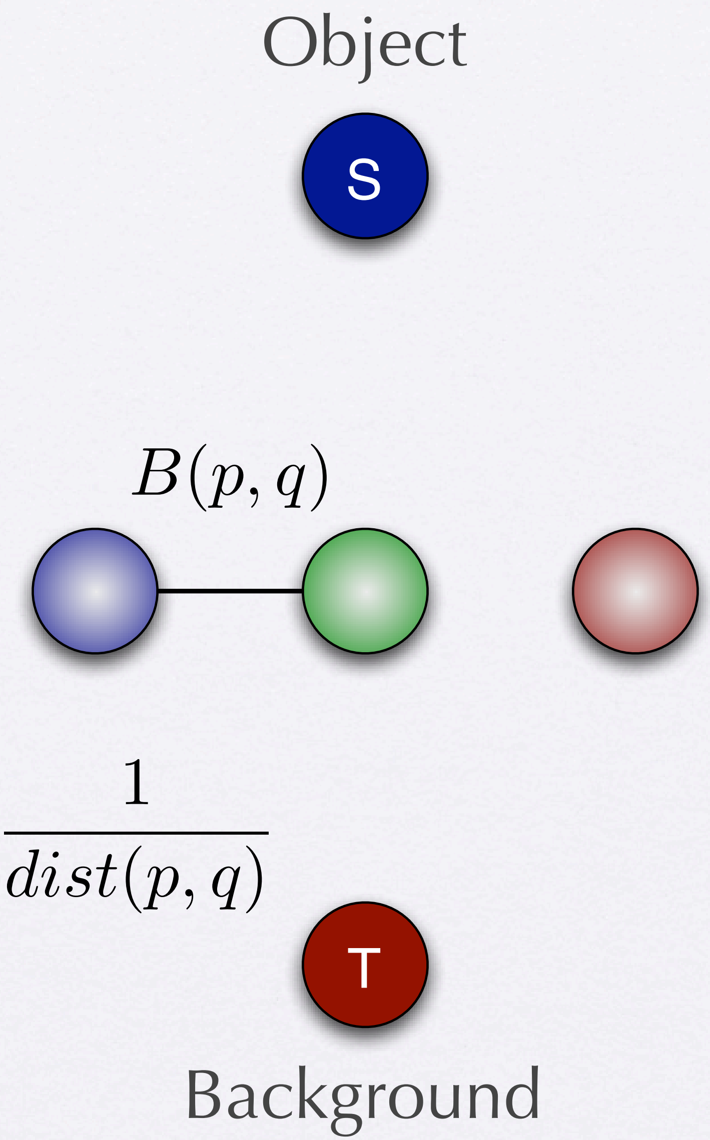
$$E(L) = \lambda \cdot \sum_{p \in P} R_p(L_p) + \sum_{(p,q) \in N} B(p,q) \cdot |L_p - L_q|$$



$$B(p,q) \propto \exp\left(-\frac{(I_p - I_q)^2}{2\sigma^2}\right) \cdot \frac{1}{dist(p,q)}$$

Graph construction: cost of n-links

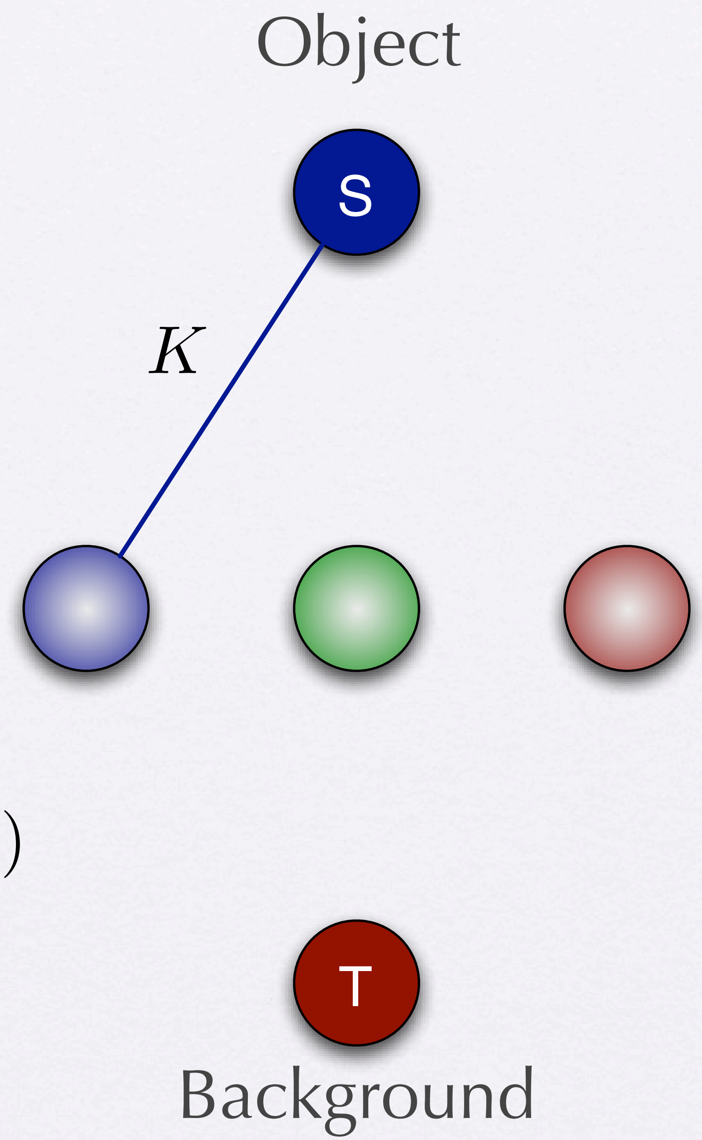
$$c(p,q) = B(p,q) \text{ if } (p,q) \in N$$



$$B(p,q) \propto \exp\left(-\frac{(I_p - I_q)^2}{2\sigma^2}\right) \cdot \frac{1}{dist(p,q)}$$

Graph construction: cost of t-link (p, S)

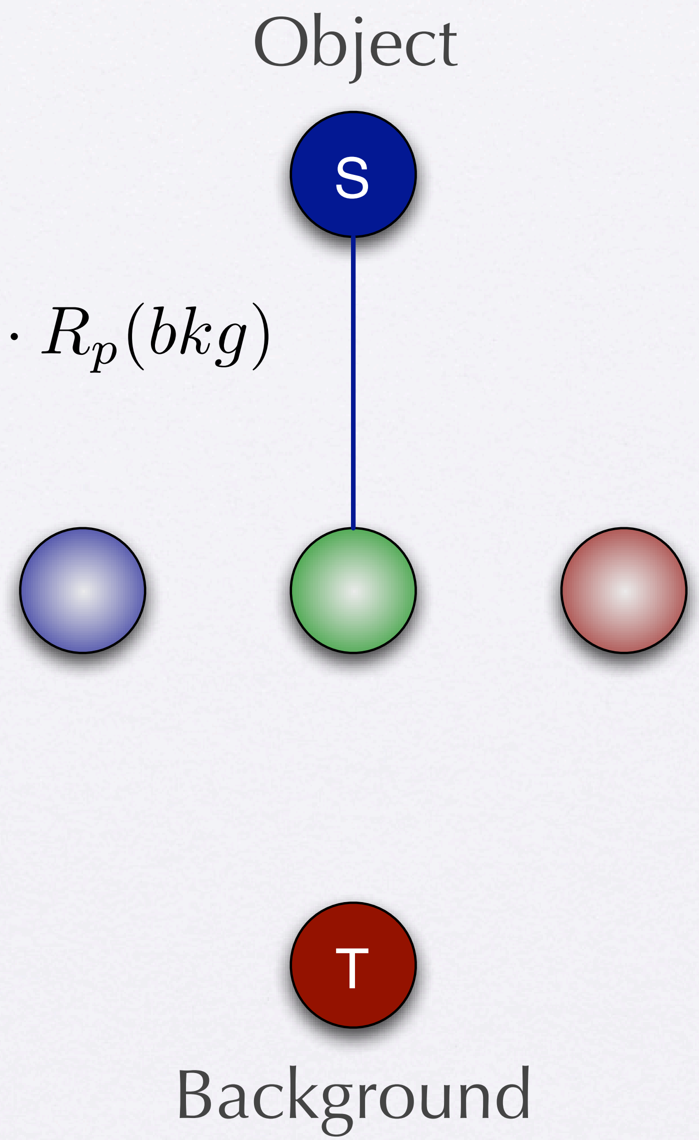
If $p \in \mathcal{O}$ then $c(p, q) = K$



$$K = 1 + \max_{p \in P} \sum_{q:(p,q) \in N} B(p, q)$$

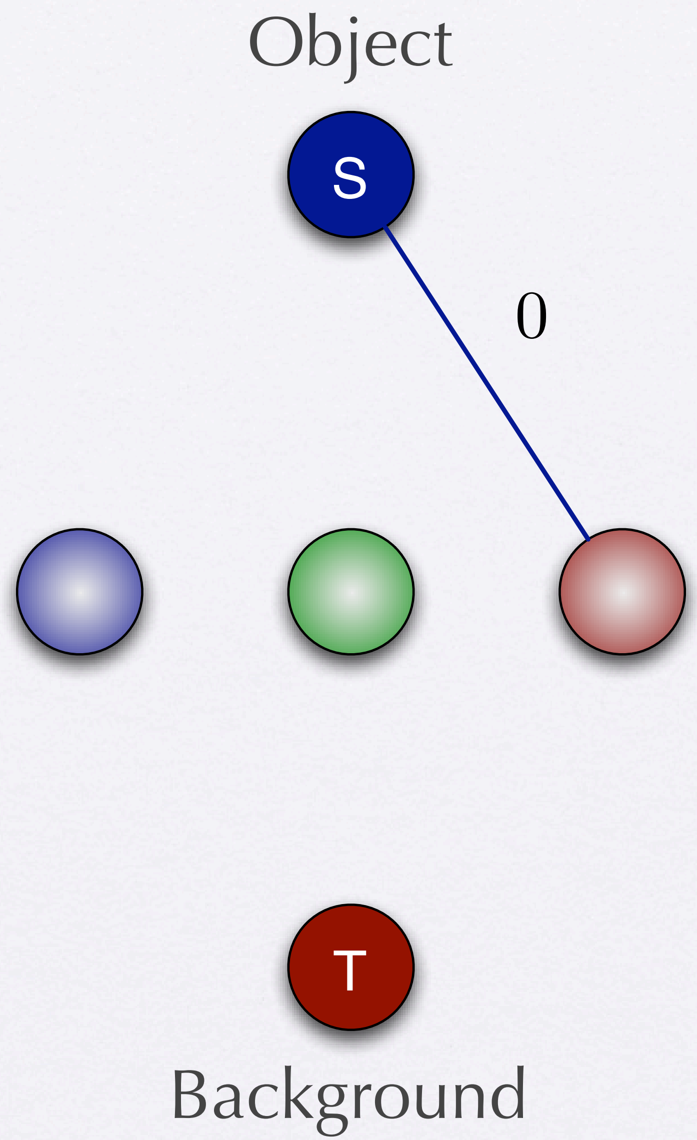
Graph construction: cost of t-link (p, S)

If $p \notin \mathcal{O} \cup \mathcal{B}$ then $c(p, q) = \lambda \cdot R_p(bkg)$



Graph construction: cost of t-link (p, S)

If $p \in \mathcal{B}$ then $c(p, q) = 0$

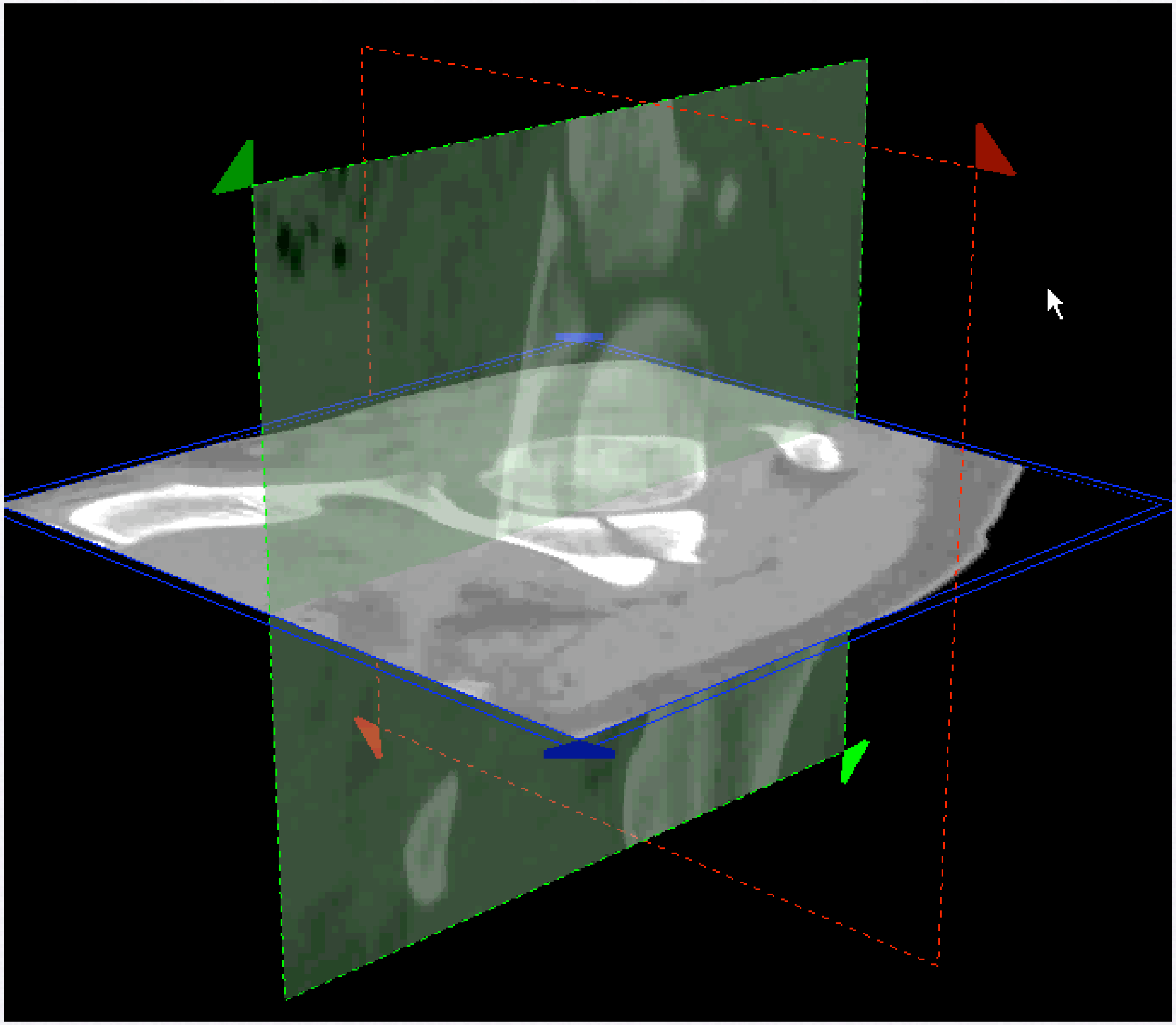


Claim: min-cut of graph minimizes energy

- Min-cut on G is a feasible cut
- Each feasible cut has a unique binary segmentation
- Segmentation associated with min-cut that satisfies user defined constraints minimizes the energy function

Summary of max-flow/min-cut algorithms

- Augmenting paths (Ford and Fulkerson)
- Push-relabel (Goldberg and Tarjan)
- Their implementation (see [2])



From [3]

Citations

- [1] Kolmogorov, V. and Zabih, R. What energy functions can be minimized via graph cuts? *IEEE Trans. Pattern Analysis and Machine Intelligence,* vol. 26, no. 2, pp. 147-159, February 2004.

- [2] Boykov, Y. and Kolmogorov, V. An experimental comparison of min-cut/max-flow algoritms for energy minimization in computer vision. *IEEE Trans. Pattern Analysis and Machine Intelligence,* vol. 26, no. 9, pp. 1124-1137, September 2004.

- [3] Boykov, Y., Torr, T. and Zabih, R. Tutorial on Discrete Optimization Methods in Computer Vision. ECCV 2004