

CSE 245: Computer Aided Circuit Simulation and Verification

Fall 2004, Sep 28

Lecture 2:

Dynamic Linear System

Outline

□ Time Domain Analysis

- State Equations
- RLC Network Analysis by Taylor Expansion
- Impulse Response in time domain

□ Frequency Domain Analysis

- From time domain to Frequency domain
- Correspondence between time domain and frequency domain
- Serial expansion of $(s\mathbf{I}-\mathbf{A})^{-1}$

Outline (Cont')

- Model Order Reduction
 - Moments
 - Passivity, Stability and Realizability
- Symbolic Analysis
 - Y-Delta Transformation
 - BDD Analysis

State of a system

- The *state* of a system is a set of data, the value of which at any time t , together with the input to the system at time t , determine uniquely the value of any network variable at time t .
- We can express the *state* in vector form

$$\mathbf{x} = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \cdot \\ \cdot \\ \cdot \\ x_k(t) \end{bmatrix}$$

Where $x_i(t)$ is the *state variables* of the system

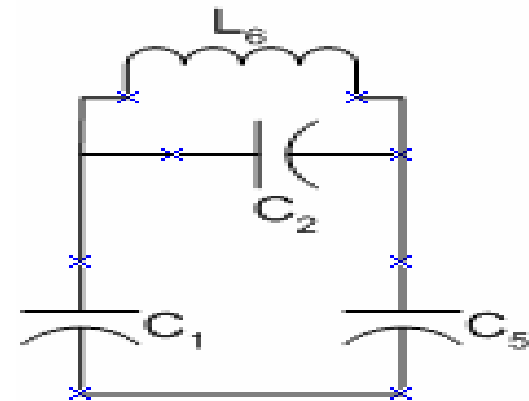
State Variable

□ How to Choose State Variable?

- The knowledge of the instantaneous values of all branch currents and voltages determines this instantaneous **state**
- But NOT ALL these values are required in order to determine the instantaneous **state**, some can be derived from others.
- choose capacitor voltages and inductor currents as the state variables! But not all of them are chosen

Degenerate Network

- A network that has a cut-set composed only of inductors and/or current sources or a loop that contains only of capacitors and/or voltage sources is called a *degenerate network*
- Example: The following network is a degenerate network since C_1 , C_2 and C_5 form a degenerate capacitor loop

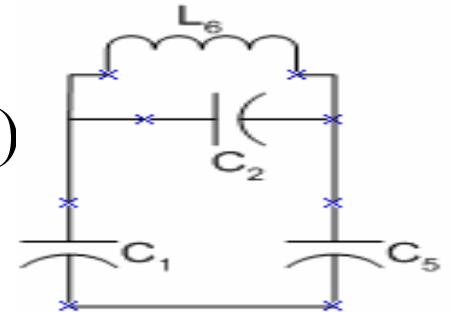


Degenerate Network

- In a degenerated network, not all the capacitors and inductors can be chosen as state variables since there are some redundancy
- On the other hand, we choose all the capacitor voltages and inductors currents as state variable in a nondegenerate network
- We will give an example of how to choose state variable in the following section

Order of Circuit

- $n = b_{LC} - n_C - n_L$
 - n the order of circuit, total number of independent state variables
 - b_{LC} total number of capacitors and inductors in the network
 - n_C number of degenerate loops (C-E loops)
 - n_L number of degenerate cut-sets (L-J cut-sets)
 - $n = 4 - 1 = 3$
-
- In a nondegenerate network, n equals to the total number of energy storage elements



State Equations

- Linear system of ordinary differential equations

$$\frac{dx}{dt} = Ax(t) + Bu(t)$$

Diagram illustrating the state equation:

- State**: An arrow points from the word "State" to the variable $x(t)$ in the equation.
- Input**: An arrow points from the word "Input" to the variable $u(t)$ in the equation.

$$y(t) = Qx(t) + Du(t)$$

Diagram illustrating the output equation:

- Output**: An arrow points from the word "Output" to the variable $y(t)$ in the equation.

State Equation for RLC Circuits

- The state equation is of the form

$$\begin{bmatrix} \mathbf{C} & 0 \\ 0 & \mathbf{L} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{v}}_t \\ \dot{\mathbf{i}}_l \end{bmatrix} = - \begin{bmatrix} \mathbf{Y} & \mathbf{E} \\ -\mathbf{E}^T & \mathbf{R} \end{bmatrix} \begin{bmatrix} \mathbf{v}_t \\ \mathbf{i}_l \end{bmatrix} + \mathbf{P}\mathbf{u}$$

- Or

$$\mathbf{M} \dot{\mathbf{x}}(t) = \mathbf{G}\mathbf{x}(t) + \mathbf{P}\mathbf{u}(t)$$

- $\mathbf{v}_t$: voltage in the trunk, capacitor voltage
- $\mathbf{i}_l$: current in the loop, inductor current.
- $\mathbf{Y}$ and $\mathbf{R}$ are the admittance matrix and impedance matrix of cut-set and mesh
- $\mathbf{E}$ covers the co-tree branches in the cut-set
- $-\mathbf{E}^T$ covers the tree trunks in the mesh analysis

State Equations

$$\mathbf{M}\dot{\mathbf{x}}(t) = \mathbf{G}\mathbf{x}(t) + \mathbf{P}\mathbf{u}(t)$$

- If we shift the matrix $\mathbf{M}$ to the right hand side, we have

$$\dot{\mathbf{x}}(t) = \mathbf{M}^{-1}\mathbf{A}\mathbf{x}(t) + \mathbf{M}^{-1}\mathbf{B}\mathbf{u}(t)$$

- Let $\mathbf{A} = \mathbf{M}^{-1}\mathbf{G}$ and $\mathbf{B} = \mathbf{M}^{-1}\mathbf{P}$, we have the state equation

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$

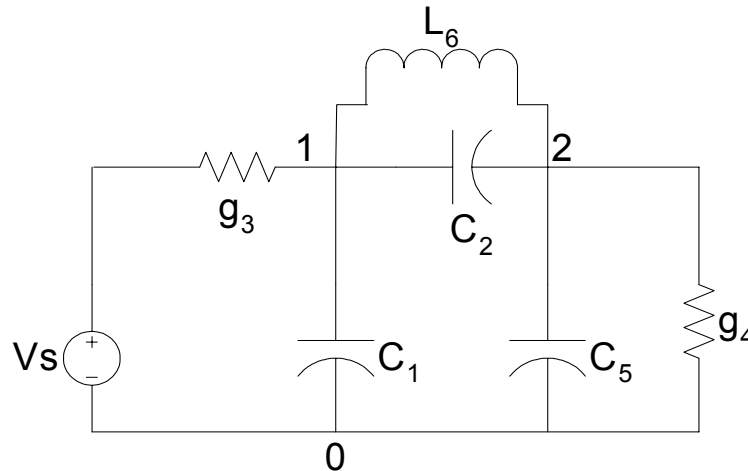
- Together with the output equation

$$\mathbf{y}(t) = \mathbf{Q}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)$$

- are called the State Equations of the linear system

RLC Network Analysis

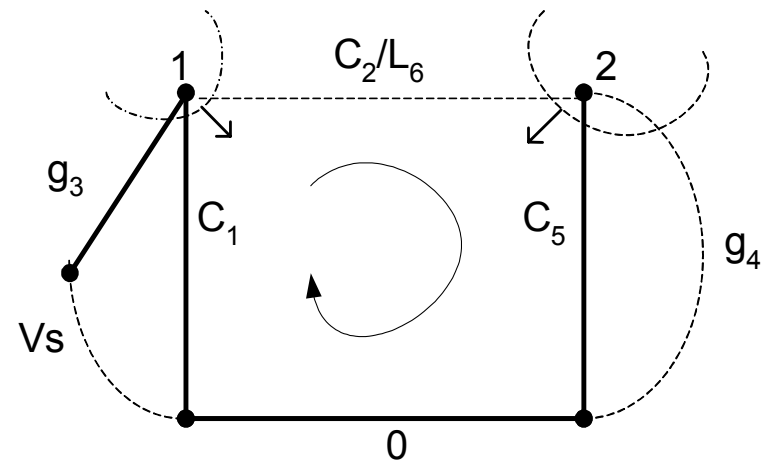
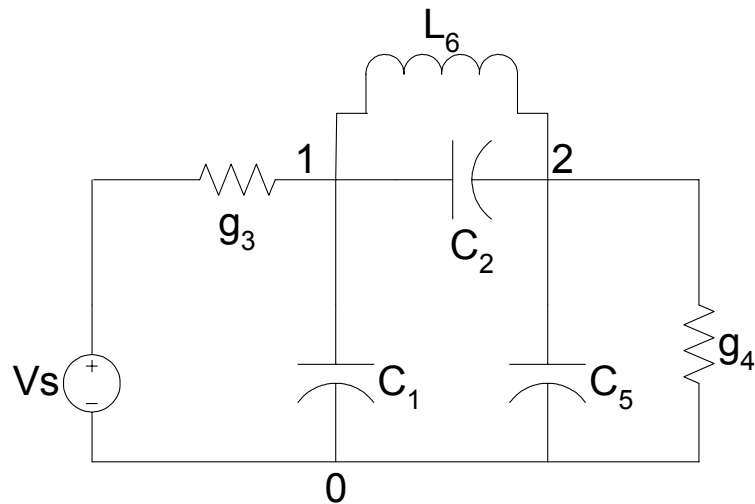
- A given RLC network



- Degenerate Network, Choose only voltages of C_1 and C_5 , current of L_6 as our state variable

Tree Structure

- Take into tree as many capacitors as possible and,
- as less inductors as possible
- Resistors can be chosen as either tree branches or co-tree branches



Linear State Equation

- By a mixed cut-set and mesh analysis, consider capacitor cut-sets and inductor loops only. we can write the linear state equation as follows

$$\begin{array}{l} \text{Cut-set KCL} \\ \text{Cut-set KCL} \\ \text{Loop KVL} \end{array} \begin{bmatrix} C_1 + C_2 & -C_2 & 0 \\ -C_2 & C_2 + C_5 & 0 \\ 0 & 0 & L_6 \end{bmatrix} \begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \\ \dot{i}_6 \end{bmatrix} = - \begin{bmatrix} g_3 & 0 & 1 \\ 0 & g_4 & -1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ i_6 \end{bmatrix} + \begin{bmatrix} g_3 \\ 0 \\ 0 \end{bmatrix} V_s$$

$$\mathbf{M} \dot{\mathbf{x}}(t) = \mathbf{G}\mathbf{x}(t) + \mathbf{P}\mathbf{u}(t)$$

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Solving RCL Equation by Taylor Expansion(1)

- General Circuit Equation

$$\dot{X} = AX + BU$$

- Consider homogeneous form first

$$\dot{X} = AX \longrightarrow X = e^{At} X_0$$

$$\text{and } e^{At} = I + \frac{At}{1!} + \frac{A^2 t^2}{2!} + \dots + \frac{A^k t^k}{k!} + \dots$$

Q: How to Compute A^k ?

Solving RCL Equation by Taylor Expansion (2)

- Assume A has non-degenerate eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_k$ and corresponding linearly independent eigenvectors $X_1, X_2, \dots, X_k$, then A can be decomposed as

$$A = X \Lambda X^{-1}$$

where $\Lambda = \begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & \vdots \\ \vdots & \dots & \ddots & \vdots \\ 0 & \dots & \dots & \lambda_k \end{bmatrix}$ and $X = [X_1, X_2, \dots, X_k]$

Solving RCL Equation by Taylor Expansion (3)

□ What's the implication then?

$$A = X\Lambda X^{-1} \Rightarrow A^2 = X\Lambda^2 X^{-1}$$

$$\Rightarrow e^{At} = X e^{\Lambda t} X^{-1} \text{ where } e^{\Lambda t} = \begin{bmatrix} e^{\lambda_1} & 0 & \dots & 0 \\ 0 & e^{\lambda_2} & \dots & \vdots \\ \vdots & \dots & \ddots & \vdots \\ 0 & \dots & \dots & e^{\lambda_k} \end{bmatrix}$$

□ To compute the eigenvalues:

$$\begin{aligned} \det(\lambda I - A) &= \lambda^n + c_{n-1}\lambda^{n-1} + \dots + c_0 \\ &= (\lambda - p_0)(\lambda^2 + p_1\lambda + p_2)\dots(\dots) = 0 \end{aligned}$$

real
eigenvalue

Conjugative
Complex
eigenvalue

Solving RCL Equation by Taylor Expansion (4)

In the previous example
$$\begin{bmatrix} v_2 \\ i_1 \end{bmatrix} = e^{At} X_0 = e^{\begin{bmatrix} 0 & 1/c \\ -1/l & -r/l \end{bmatrix} t} \begin{bmatrix} v_2(0) \\ i_1(0) \end{bmatrix}$$

Let $c=r=l=1$, we have
$$A = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} = X \Lambda X^{-1} = X \begin{bmatrix} \lambda_+ & 0 \\ 0 & \lambda_- \end{bmatrix} X^{-1}$$

where
$$\lambda_+ = \frac{-1 + \sqrt{3}j}{2} \quad \lambda_- = \frac{-1 - \sqrt{3}j}{2}$$
$$X = \frac{j}{\sqrt{3}} \begin{bmatrix} \frac{-1 - \sqrt{3}j}{2} & -1 \\ \frac{1 - \sqrt{3}j}{2} & 1 \end{bmatrix} \quad X^{-1} = \begin{bmatrix} 1 & 1 \\ \frac{-1 + \sqrt{3}j}{2} & \frac{-1 - \sqrt{3}j}{2} \end{bmatrix}$$

hence
$$e^{At} = X \begin{bmatrix} e^{\lambda_+ t} & 0 \\ 0 & e^{\lambda_- t} \end{bmatrix} X^{-1}$$

Solving RCL Equation by Taylor Expansion (5)

- What if matrix A has degenerated eigenvalues? **Jordan decomposition !**

$$A = XJX^{-1}$$

J is in the Jordan Canonical form

$$\begin{bmatrix} \lambda_1 & 1 & 0 & \cdots & 0 \\ 0 & \lambda_1 & 1 & \ddots & 0 \\ 0 & 0 & \lambda_1 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 1 \\ 0 & 0 & 0 & \cdots & \lambda_1 \\ & & & \ddots & \\ & & & & \lambda_k & 1 & 0 & \cdots & 0 \\ & & & & 0 & \lambda_k & 1 & \ddots & 0 \\ & & & & 0 & 0 & \lambda_k & \ddots & 0 \\ & & & & \vdots & \ddots & \ddots & \ddots & 1 \\ & & & & 0 & 0 & 0 & \cdots & \lambda_k \end{bmatrix}$$

And still $e^{At} = Xe^{Jt}X^{-1}$

Jordan Decomposition

$$J = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix} \Rightarrow e^{Jt} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix} t + \dots = \begin{bmatrix} e^{\lambda t} & te^{\lambda t} \\ 0 & e^{\lambda t} \end{bmatrix}$$

similarly

$$J = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix} \Rightarrow e^{Jt} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix} t + \dots = \begin{bmatrix} e^{\lambda t} & te^{\lambda t} & \frac{t^2}{2!}e^{\lambda t} \\ 0 & e^{\lambda t} & te^{\lambda t} \\ 0 & 0 & e^{\lambda t} \end{bmatrix}$$

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Response in time domain

- We can solve the state equation and get the closed form expression

$$\begin{aligned}\mathbf{x}(t) &= e^{At} \int_{t_0}^t e^{-A\tau} \mathbf{B} \mathbf{u}(\tau) d\tau + e^{A(t-t_0)} \mathbf{x}(t_0) \\ &= \int_{t_0}^t e^{A(t-\tau)} \mathbf{B} \mathbf{u}(\tau) d\tau + e^{A(t-t_0)} \mathbf{x}(t_0) \\ &= (e^{At}) \mathbf{B} * \mathbf{u}(t) + e^{A(t-t_0)} \mathbf{x}(t_0)\end{aligned}$$

Note: * denotes convolution

- The output equation can be expressed as

$$\begin{aligned}\mathbf{y}(t) &= \mathbf{Q} \int_{t_0}^t e^{A(t-\tau)} \mathbf{B} \mathbf{u}(\tau) d\tau + \mathbf{Q} e^{A(t-t_0)} \mathbf{x}(t_0) + \mathbf{D} \mathbf{u}(t) \\ &= \int_{t_0}^t \{ \mathbf{Q} e^{A(t-\tau)} \mathbf{B} + \mathbf{D} \delta(t-\tau) \} \mathbf{u}(\tau) d\tau + \mathbf{Q} e^{A(t-t_0)} \mathbf{x}(t_0) \\ &= (\mathbf{Q} e^{A(t-\tau)} \mathbf{B} + \mathbf{D} \delta(t)) * \mathbf{u}(t) + \mathbf{Q} e^{A(t-t_0)} \mathbf{x}(t_0)\end{aligned}$$

Impulse Response

- The *Impulse Response* of a system is defined as the Zero State Response resulting from an impulse excitation
- Thus, in the output equation, replace $\mathbf{u}(t)$ by the impulse function $\delta(t)$, and let $\mathbf{x}(t_0)=0$ we have

$$\mathbf{h}(t) = \mathbf{y}(t) = \mathbf{Q}e^{\mathbf{A}t} \mathbf{B}$$

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Solutions in S domain

- By solving the state equation in s domain, we have

$$\mathbf{x}(s) = (s\mathbf{I}-\mathbf{A})^{-1} \mathbf{x}(t_0) + (s\mathbf{I}-\mathbf{A})^{-1} \mathbf{B}\mathbf{u}(s)$$

$$\mathbf{y}(s) = \mathbf{Q}\mathbf{x}(s) + \mathbf{D}\mathbf{u}(s) = \mathbf{Q}(s\mathbf{I}-\mathbf{A})^{-1}(\mathbf{x}(t_0) + \mathbf{B}\mathbf{u}(s)) + \mathbf{D}\mathbf{u}(s)$$

- Suppose the network has zero state and the output vector depends only on the state vector $\mathbf{x}$, that is, $\mathbf{x}(t_0) = 0$ and $\mathbf{D} = 0$, we can derive the transfer function of the network

$$\mathbf{H}(s) = \frac{\mathbf{y}(s)}{\mathbf{u}(s)} = \mathbf{Q}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}$$

Frequency Domain Representation

**Bilateral
Laplace
Transform:**

$$x(t) \quad \Rightarrow \quad x(s) = \int_{-\infty}^{\infty} x(t)e^{-st} dt$$

**Key
Transform
Property:**

$$\frac{dx}{dt} \quad \Rightarrow \quad sx(s)$$

$$\begin{aligned} \frac{dx(t)}{dt} &= Ax(t) + Bu(t) \\ y(t) &= Qx(t) \end{aligned} \quad \Rightarrow \quad \begin{aligned} sx(s) &= Ax(s) + Bu(s) \\ y(s) &= Qx(s) \end{aligned}$$

System Transfer Function

$$sx(s) = Ax(s) + Bu(s)$$

$$y(s) = Qx(s)$$



**Express $y(s)$ as a
function of $u(s)$**

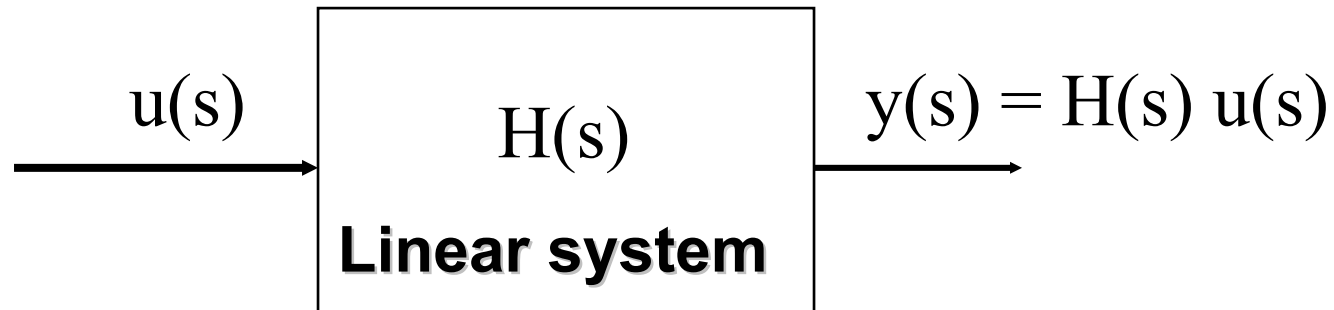
$$y(s) = \underbrace{-Q(sI - A)^{-1}B}_{H(s)}u(s)$$

Transfer Function: $H(s)$

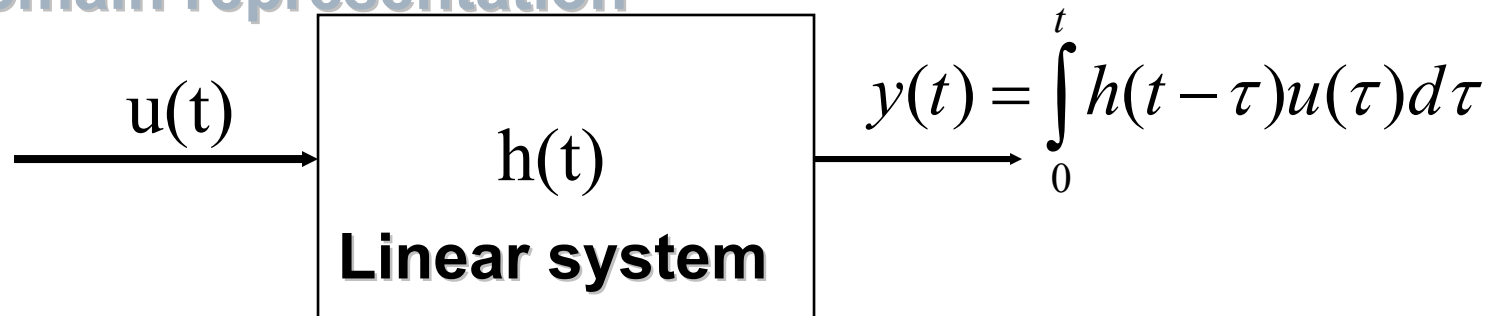
Transfer Function

Time Domain Impulse Response

Frequency domain representation



Time domain representation



The transfer function $H(s)$ is the Laplace Transform of the impulse response $h(t)$

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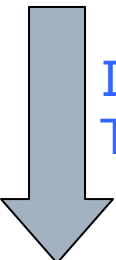
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Correspondence between time domain and frequency domain

- We can derive the time domain solutions of the network from the s domain solutions by inverse Laplace Transformation of the s domain solutions.

State Equations
in **S domain**

$$\begin{cases} s\mathbf{x}(s) - \mathbf{x}(t_0) = \mathbf{A}\mathbf{x}(s) + \mathbf{B}\mathbf{u}(s) \\ \mathbf{y}(s) = \mathbf{Q}\mathbf{x}(s) + \mathbf{D}\mathbf{u}(s) \end{cases}$$

 Inverse Laplace Transform

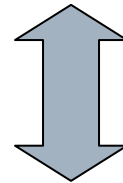
State Equations
in **time Domain**

$$\begin{cases} \mathbf{x}(t) = L^{-1}[(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{x}(t_0) + (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}\mathbf{u}(s)] \\ \quad = L^{-1}[(s\mathbf{I} - \mathbf{A})^{-1}]\mathbf{x}(t_0) + L^{-1}[(s\mathbf{I} - \mathbf{A})^{-1}]\mathbf{B} * \mathbf{u}(t) \\ \mathbf{y}(t) = L^{-1}[\mathbf{Q}(s\mathbf{I} - \mathbf{A})^{-1}(\mathbf{x}(t_0) + \mathbf{B}\mathbf{u}(s)) + \mathbf{D}\mathbf{u}(s)] \\ \quad = \mathbf{Q} L^{-1}[(s\mathbf{I} - \mathbf{A})^{-1}]\mathbf{x}(t_0) + \{\mathbf{Q}L^{-1}[(s\mathbf{I} - \mathbf{A})^{-1}]\mathbf{B} + \mathbf{D}\delta(t)\} * \mathbf{u}(s) \end{cases}$$

Correspondence between time domain and frequency domain

Solution from
time domain
analysis

$$\begin{aligned}\mathbf{x}(t) &= (\mathbf{e}^{\mathbf{A}t})\mathbf{B}*\mathbf{u}(t) + \mathbf{e}^{\mathbf{A}(t-t_0)}\mathbf{x}(t_0) \\ \mathbf{y}(t) &= (\mathbf{Q}\mathbf{e}^{\mathbf{A}(t-\tau)}\mathbf{B} + \mathbf{D}\delta(t)) * \mathbf{u}(t) + \mathbf{Q}\mathbf{e}^{\mathbf{A}(t-t_0)}\mathbf{x}(t_0)\end{aligned}$$



Solution by
inverse Laplace
transform

$$\begin{aligned}\mathbf{x}(t) &= \mathcal{L}^{-1}[(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{x}(t_0) + (s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}\mathbf{u}(s)] \\ &= \mathcal{L}^{-1}[(s\mathbf{I}-\mathbf{A})^{-1}]\mathbf{x}(t_0) + \mathcal{L}^{-1}[(s\mathbf{I}-\mathbf{A})^{-1}]\mathbf{B}*\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathcal{L}^{-1}[\mathbf{Q}(s\mathbf{I}-\mathbf{A})^{-1}(\mathbf{x}(t_0) + \mathbf{B}\mathbf{u}(s)) + \mathbf{D}\mathbf{u}(s)] \\ &= \mathbf{Q}\mathcal{L}^{-1}[(s\mathbf{I}-\mathbf{A})^{-1}]\mathbf{x}(t_0) + \{\mathbf{Q}\mathcal{L}^{-1}[(s\mathbf{I}-\mathbf{A})^{-1}]\mathbf{B} + \mathbf{D}\delta(t)\} * \mathbf{u}(s)\end{aligned}$$

□ $(s\mathbf{I}-\mathbf{A})^{-1} \longleftrightarrow \mathbf{e}^{\mathbf{A}t}$

□ multiplication of $\mathbf{u}(s)$ in s domain corresponds to the convolution in time domain

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Serial expansion of $(s\mathbf{I}-\mathbf{A})^{-1}$

- When $s \rightarrow 0$ we can write $(s\mathbf{I}-\mathbf{A})^{-1}$ as

$$(s\mathbf{I}-\mathbf{A})^{-1} = -\mathbf{A}^{-1}(\mathbf{I} - s\mathbf{A}^{-1}) = -\mathbf{A}^{-1}(\mathbf{I} + s\mathbf{A}^{-1} + s^2\mathbf{A}^{-2} + \dots + s^k\mathbf{A}^{-k} + \dots)$$

- Thus, the transfer function can be written as

$$\mathbf{H}(s) = \mathbf{Q}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B} = -\mathbf{Q}\mathbf{A}^{-1}(\mathbf{I} + s\mathbf{A}^{-1} + s^2\mathbf{A}^{-2} + \dots + s^k\mathbf{A}^{-k} + \dots)\mathbf{B}$$

- When $s \rightarrow \infty$ we can write $(s\mathbf{I}-\mathbf{A})^{-1}$ as

$$(s\mathbf{I}-\mathbf{A})^{-1} = s^{-1}(\mathbf{I} - s^{-1}\mathbf{A})^{-1} = s^{-1}(\mathbf{I} + s^{-1}\mathbf{A} + s^{-2}\mathbf{A}^2 + \dots + s^{-k}\mathbf{A}^k + \dots)$$

- The transfer function can be written as

$$\mathbf{H}(s) = \mathbf{Q}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B} = s^{-1}(\mathbf{I} + s^{-1}\mathbf{A} + s^{-2}\mathbf{A}^2 + \dots + s^{-k}\mathbf{A}^k + \dots)\mathbf{B}$$

Matrix Decomposition

- Assume A has non-degenerate eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_k$ and corresponding linearly independent eigenvectors $X_1, X_2, \dots, X_k$, then A can be decomposed as

$$A = X\Lambda X^{-1}$$

where $\Lambda = \begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & \vdots \\ \vdots & \dots & \ddots & \vdots \\ 0 & \dots & \dots & \lambda_k \end{bmatrix}$ and $X = [X_1, X_2, \dots, X_k]$

Matrix Decomposition

- Then we can write $(s\mathbf{I}-\mathbf{A})^{-1}$ in the following form

$$(s\mathbf{I}-\mathbf{A})^{-1} = (\mathbf{S}\mathbf{I} - \mathbf{X}\mathbf{\Lambda}\mathbf{X}^{-1})^{-1} = \mathbf{X}^{-1}(\mathbf{s}\mathbf{I} - \mathbf{\Lambda})^{-1}\mathbf{X} = \mathbf{X}^{-1} \begin{bmatrix} \frac{1}{s-\lambda_1} & & & \\ & \frac{1}{s-\lambda_2} & & \\ & & \ddots & \\ & & & \frac{1}{s-\lambda_n} \end{bmatrix} \mathbf{X}$$

- $(s\mathbf{I}-\mathbf{A})^{-1}$ in s domain corresponds to the exponential function $e^{\mathbf{A}t}$ in time domain, we can write $e^{\mathbf{A}t}$ as

$$e^{\mathbf{A}t} = \mathbf{X}^{-1} \begin{bmatrix} e^{\lambda_1 t} & & & \\ & e^{\lambda_2 t} & & \\ & & \ddots & \\ & & & e^{\lambda_n t} \end{bmatrix} \mathbf{X}$$