

Computer-Aided Circuit Simulation and Verification

CSE245 – Fall 2004

Professor: Chung-Kuan Cheng

Administration

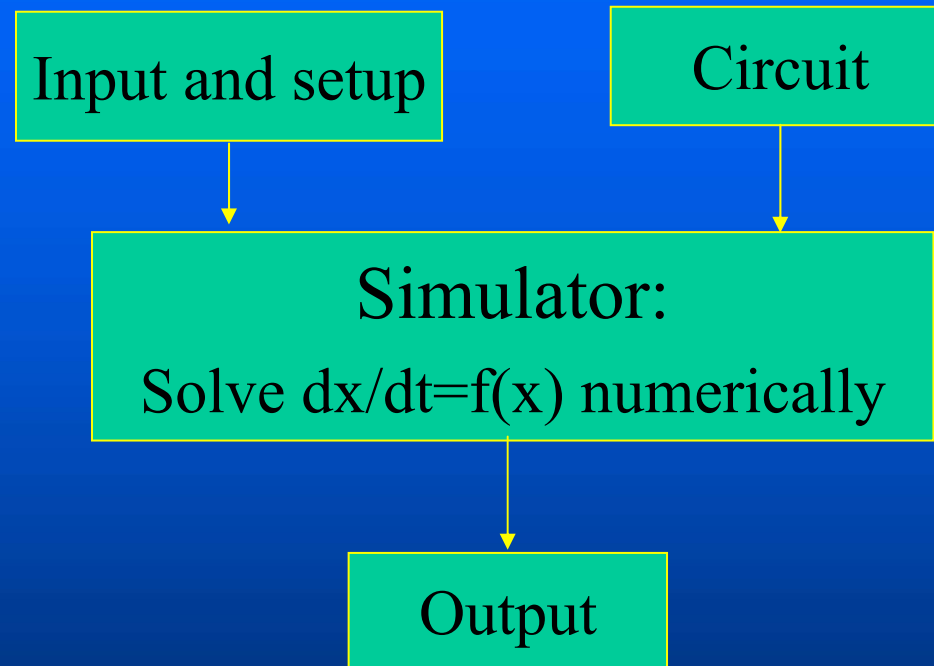
- Lectures: 5:00pm ~ 6:20pm TTH HSS 2152
- Office Hours: 4:00pm ~ 4:45pm TTH APM 4256
- Textbook

Electronic Circuit and System Simulation Methods

T.L. Pillage, R.A. Rohrer, C. Visweswariah, McGraw-Hill

- TA: Zhengyong (Simon) Zhu (zzhu@cs.ucsd.edu)

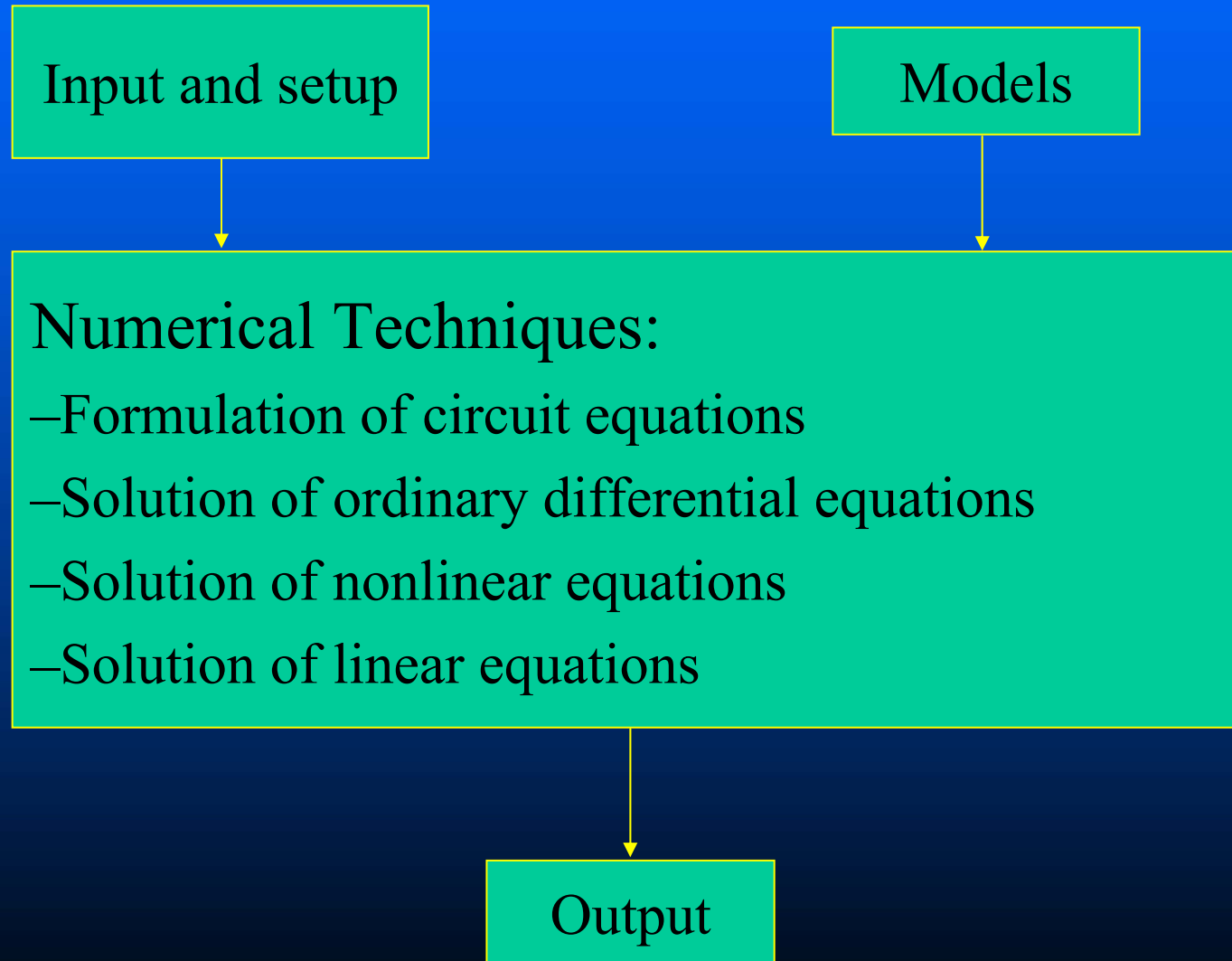
Circuit Simulation



Types of analysis:

- DC Analysis
- DC Transfer curves
- Transient Analysis
- AC Analysis, Noise, Distortion, Sensitivity

Program Structure (a closer look)



CSE245: Course Outline

- **Formulation**
 - RLC Linear, Nonlinear Components, Transistors, Diodes
 - Incident Matrix
 - Nodal Analysis, Modified Nodal Analysis
 - K Matrix
- **Dynamic Linear System**
 - S domain analysis, Impulse Response
 - Taylor's expansion
 - Moments, Passivity, Stability, Realizability
 - Symbolic analysis, Y-Delta, BDD analysis
- **Matrix Solver**
 - LU, KLU, reordering
 - Mutigrid, PCG, GMRES

CSE245: Course Outline (Cont')

- **Integration**
 - Forward Euler, Backward Euler, Trapezoidal Rule
 - Explicit and Implicit Method, Prediction and Correction
 - Equivalent Circuit
 - Errors: Local error, Local Truncation Error, Global Error
 - A-Stable
 - Alternating Direction Implicit Method
- **Nonlinear System**
 - Newton Raphson, Line Search
- **Transmission Line, S-Parameter**
 - FDTD: equivalent circuit, convolution
 - Frequency dependent components
- **Sensitivity**
- **Mechanical, Thermal, Bio Analysis**

Lecture 1: Formulation

- KCL/KVL
- Sparse Tableau Analysis
- Nodal Analysis, Modified Nodal Analysis

*some slides borrowed from Berkeley EE219 Course

Formulation of Circuit Equations

- Unknowns

- B branch currents (i)
- N node voltages (e)
- B branch voltages (v)

- Equations

- N+B Conservation Laws
- B Constitutive Equations

Branch Constitutive Equations (BCE)

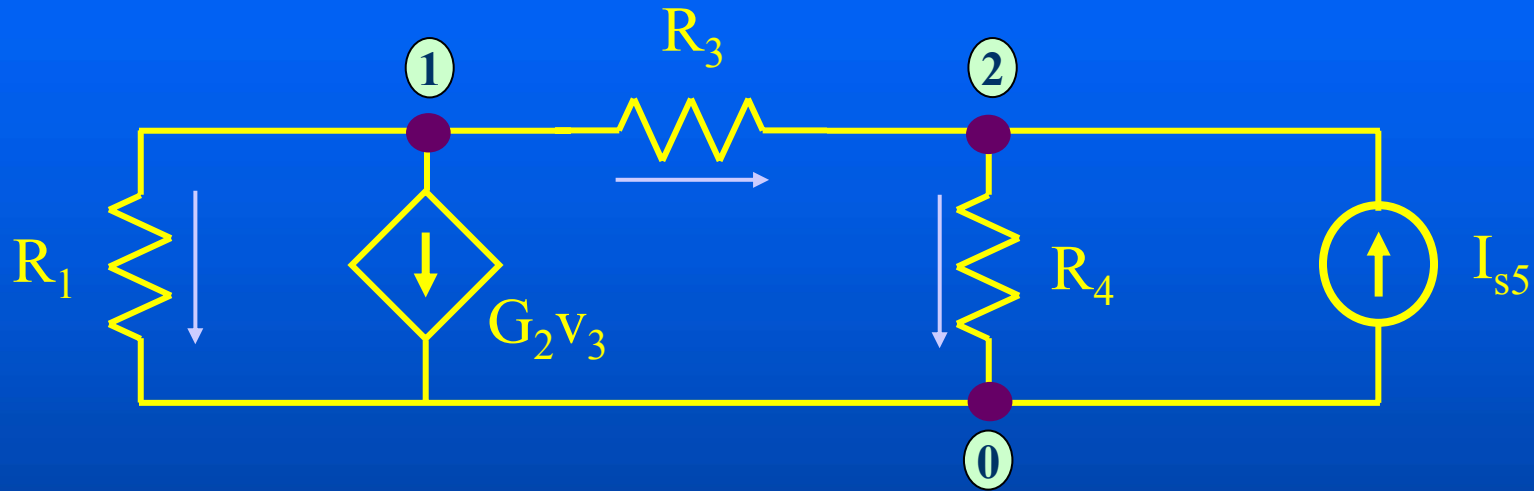
Ideal elements

Element	Branch Eqn
Resistor	$v = R \cdot i$
Capacitor	$i = C \cdot dv/dt$
Inductor	$v = L \cdot di/dt$
Voltage Source	$v = v_s, i = ?$
Current Source	$i = i_s, v = ?$
VCVS	$v_s = A_V \cdot v_c, i = ?$
VCCS	$i_s = G_T \cdot v_c, v = ?$
CCVS	$v_s = R_T \cdot i_c, i = ?$
CCCS	$i_s = A_I \cdot i_c, v = ?$

Conservation Laws

- Determined by the topology of the circuit
- **Kirchhoff's Voltage Law (KVL)**: Every circuit node has a unique voltage with respect to the reference node. The voltage across a branch e_b is equal to the difference between the positive and negative referenced voltages of the nodes on which it is incident
 - No voltage source loop
- **Kirchhoff's Current Law (KCL)**: The algebraic sum of all the currents flowing out of (or into) any circuit node is zero.
 - No Current Source Cut

Equation Formulation - KCL



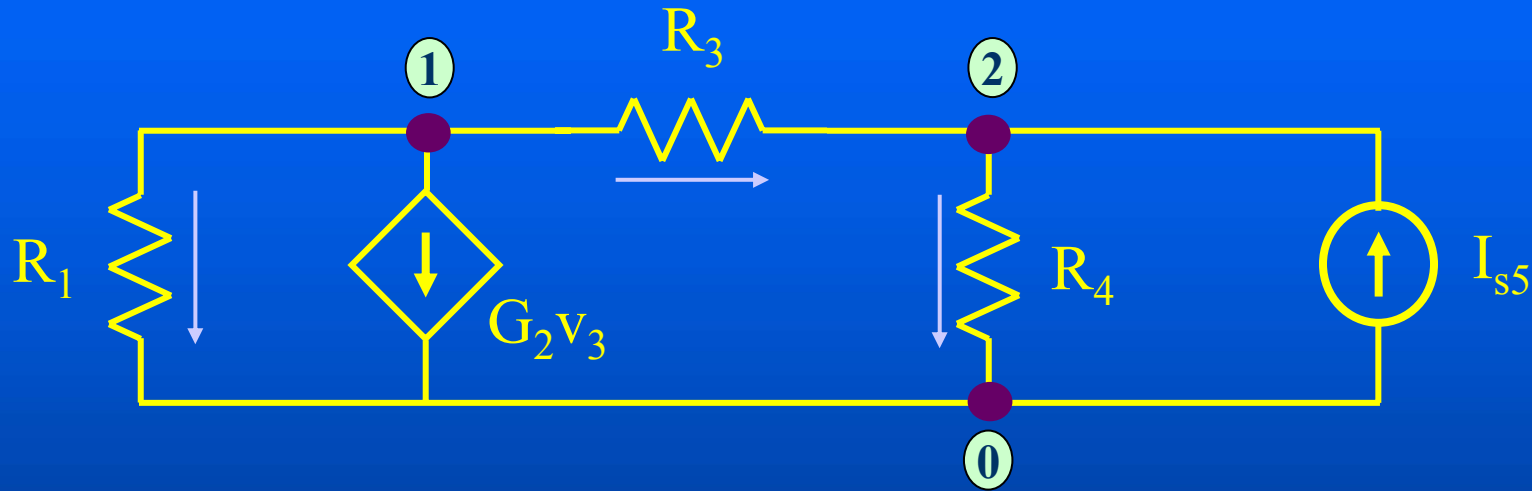
$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\mathbf{A} \mathbf{i} = \mathbf{0}$$

N equations

Kirchhoff's Current Law (KCL)

Equation Formulation - KVL



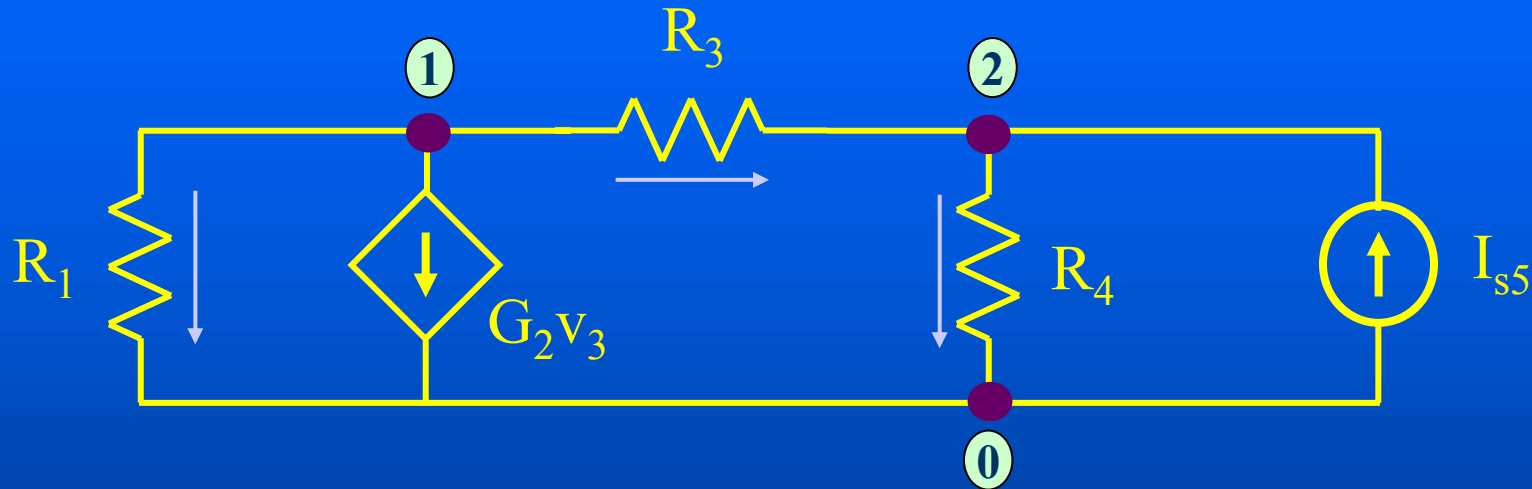
$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{v} - \mathbf{A}^T \mathbf{e} = \mathbf{0}$$

B equations

Kirchhoff's Voltage Law (KVL)

Equation Formulation - BCE

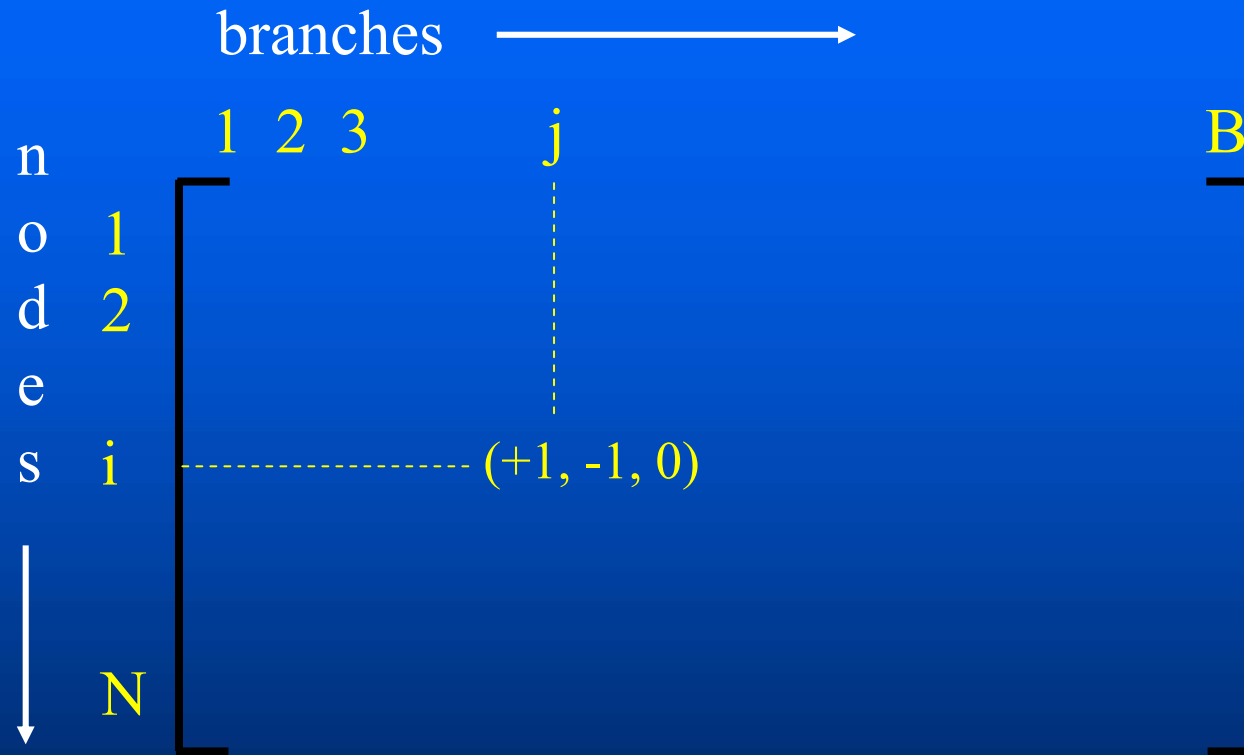


$$\begin{bmatrix} -\frac{1}{R_1} & 0 & 0 & 0 & 0 \\ 0 & 0 & -G_2 & 0 & 0 \\ 0 & 0 & -\frac{1}{R_3} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{R_4} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} + \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ i_{s5} \end{bmatrix}$$

$$\mathbf{K_v v} + \mathbf{i} = \mathbf{i_s} \quad B \text{ equations}$$

Equation Formulation

Node-Branch Incidence Matrix



$$A_{ij} = \begin{cases} +1 & \text{if node } i \text{ is terminal } + \text{ of branch } j \\ -1 & \text{if node } i \text{ is terminal } - \text{ of branch } j \\ 0 & \text{if node } i \text{ is not connected to branch } j \end{cases}$$

Equation Assembly (Stamping Procedures)

- Different ways of combining Conservation Laws and Constitutive Equations
 - **Sparse Table Analysis (STA)**
 - **Modified Nodal Analysis (MNA)**

Sparse Tableau Analysis (STA)

1. Write KCL: $Ai=0$ (N eqns)
2. Write KVL: $v - A^T e=0$ (B eqns)
3. Write BCE: $K_i i + K_v v = S$ (B eqns)

$$\begin{bmatrix} A & 0 & 0 \\ 0 & I & -A^T \\ K_i & K_v & 0 \end{bmatrix} \begin{bmatrix} i \\ v \\ e \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ S \end{bmatrix}$$

Sparse Tableau

N+2B eqns

N+2B unknowns

N = # nodes

B = # branches

Sparse Tableau Analysis (STA)

Advantages

- It can be applied to any circuit
- Eqns can be assembled directly from input data
- Coefficient Matrix is very sparse

Problem

Sophisticated programming techniques and data structures are required for time and memory efficiency

Nodal Analysis (NA)

1. Write KCL

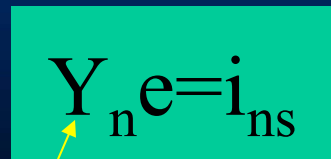
$$A \cdot i = 0 \quad (N \text{ eqns, } B \text{ unknowns})$$

2. Use BCE to relate branch currents to branch voltages

$$i = f(v) \quad (B \text{ unknowns} \rightarrow B \text{ unknowns})$$

3. Use KVL to relate branch voltages to node voltages

$$v = h(e) \quad (B \text{ unknowns} \rightarrow N \text{ unknowns})$$


$$Y_n e = i_{ns}$$

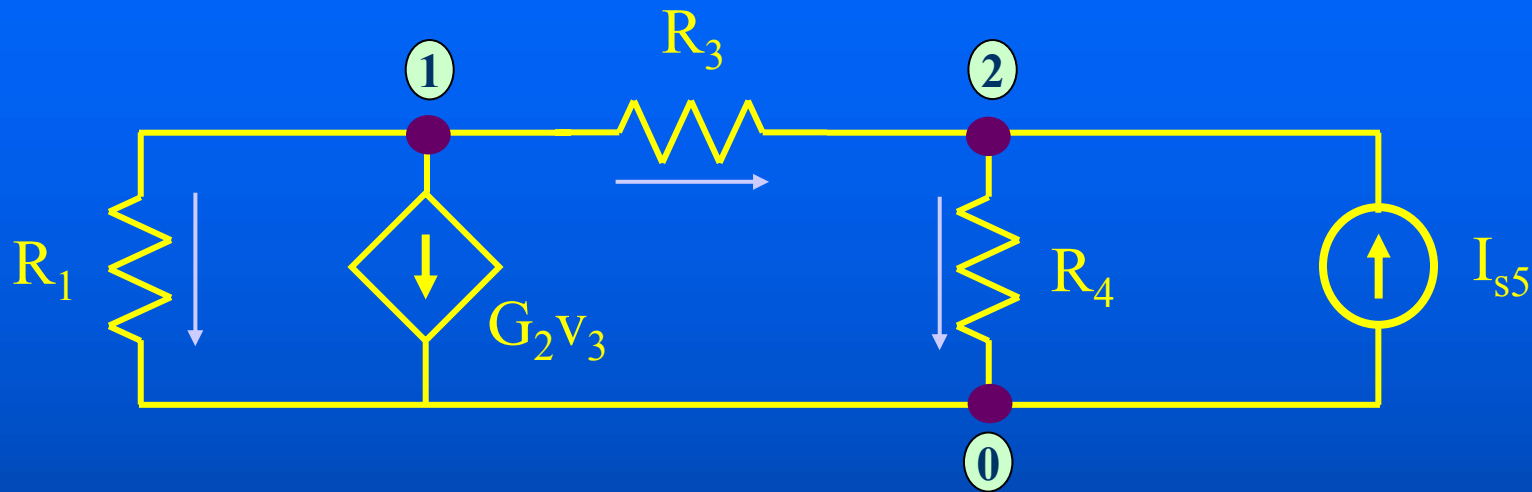
Nodal Matrix

N eqns

N unknowns

N = # nodes

Nodal Analysis - Example



1. KCL: $\sum i = 0$
2. BCE: $K_v v + i = i_s \rightarrow i = i_s - K_v v \Rightarrow A K_v v = A i_s$
3. KVL: $v = A^T e \Rightarrow A K_v A^T e = A i_s$

$$\begin{bmatrix} \frac{1}{R_1} + G_2 + \frac{1}{R_3} & -G_2 - \frac{1}{R_3} \\ -\frac{1}{R_3} & \frac{1}{R_3} + \frac{1}{R_4} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 0 \\ i_{s5} \end{bmatrix}$$

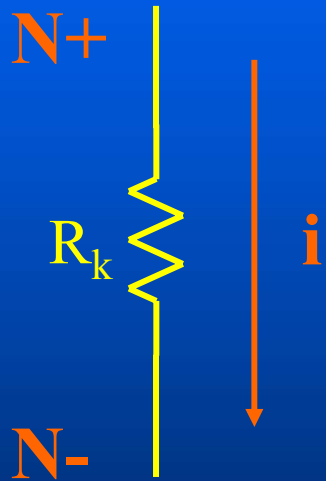
$$Y_n e = i_{ns}$$

Nodal Analysis

- Example shows NA may be derived from STA
- Better: Y_n may be obtained by direct inspection (stamping procedure)
 - Each element has an associated stamp
 - Y_n is the composition of all the elements' stamps

Nodal Analysis – Resistor “Stamp”

Spice input format: `Rk N+ N- Rkvalue`



$$\begin{array}{cc} & \begin{array}{cc} N+ & N- \end{array} \\ \begin{array}{c} N+ \\ N- \end{array} & \left[\begin{array}{cc} \frac{1}{R_k} & -\frac{1}{R_k} \\ -\frac{1}{R_k} & \frac{1}{R_k} \end{array} \right] \end{array}$$

What if a resistor is
connected to ground?

....

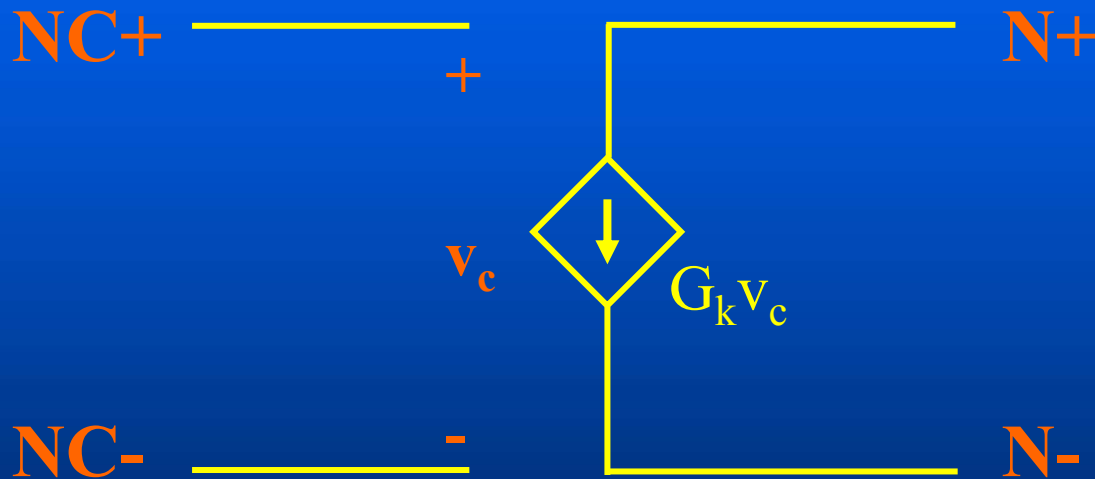
Only contributes to the
diagonal

$$\sum i_{others} + \frac{1}{R_k} (e_{N+} - e_{N-}) = \sum i_s \quad \text{KCL at node } N+$$

$$\sum i_{others} - \frac{1}{R_k} (e_{N+} - e_{N-}) = \sum i_s \quad \text{KCL at node } N-$$

Nodal Analysis – VCCS “Stamp”

Spice input format: G_k $N+$ $N-$ $NC+$ $NC-$ $G_k\text{value}$



$$\begin{array}{cc} & \begin{array}{cc} NC+ & NC- \end{array} \\ \begin{array}{c} N+ \\ N- \end{array} & \begin{bmatrix} G_k & -G_k \\ -G_k & G_k \end{bmatrix} \end{array}$$

$$\sum i_{others} + G_k (e_{NC+} - e_{NC-}) = \sum i_s$$

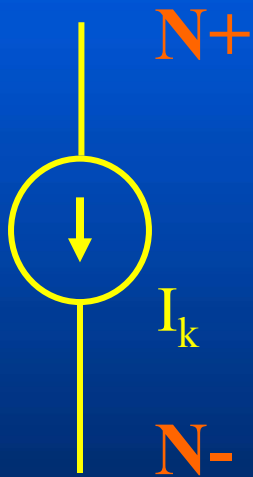
KCL at node $N+$

$$\sum i_{others} - G_k (e_{NC+} - e_{NC-}) = \sum i_s$$

KCL at node $N-$

Nodal Analysis – Current source “Stamp”

Spice input format: I_k $N+$ $N-$ I_{kvalue}



$$\begin{array}{cc} & N+ \quad N- \\ \begin{array}{c} N+ \\ N- \end{array} & \begin{bmatrix} & \\ & \end{bmatrix} = \begin{bmatrix} -I_k \\ I_k \end{bmatrix} \end{array}$$

Nodal Analysis (NA)

Advantages

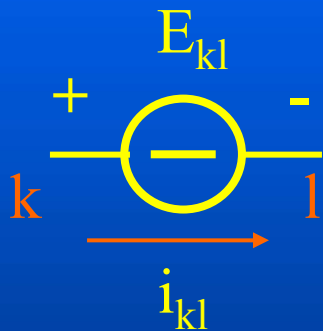
- Y_n is often diagonally dominant and symmetric
- Eqns can be assembled directly from input data
- Y_n has non-zero diagonal entries
- Y_n is sparse (not as sparse as STA) and smaller than STA: $N \times N$ compared to $(N+2B) \times (N+2B)$

Limitations

- Conserved quantity must be a function of node variable
 - Cannot handle floating voltage sources, VCVS, CCCS, CCVS

Modified Nodal Analysis (MNA)

How do we deal with independent voltage sources?

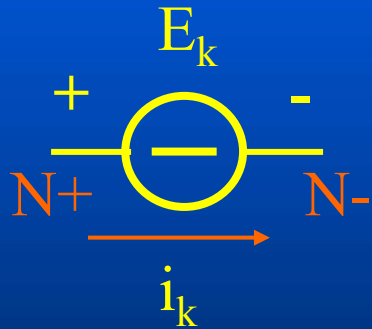


$$\begin{array}{c} k \\ l \end{array} \left[\begin{array}{cc|c} & & 1 \\ & & -1 \\ \hline 1 & -1 & 0 \end{array} \right] \begin{bmatrix} e_k \\ e_l \\ i_{kl} \end{bmatrix} = \begin{bmatrix} \\ \\ E_{kl} \end{bmatrix}$$

- i_{kl} cannot be explicitly expressed in terms of node voltages $\Rightarrow$ it has to be added as unknown (new column)
- e_k and e_l are not independent variables anymore $\Rightarrow$ a constraint has to be added (new row)

MNA – Voltage Source “Stamp”

Spice input format: V_k $N+$ $N-$ E_k value



	$N+$	$N-$	i_k	RHS
$N+$	0	0	1	$\begin{bmatrix} 0 \\ 0 \\ E_k \end{bmatrix}$
$N-$	0	0	-1	
Branch k	1	-1	0	

Modified Nodal Analysis (MNA)

How do we deal with independent voltage sources?

Augmented nodal matrix

$$\left[\begin{array}{c|c} Y_n & B \\ \hline C & 0 \end{array} \right] \begin{bmatrix} e \\ i \end{bmatrix} = MS$$


Some branch currents

In general:

$$\left[\begin{array}{c|c} Y_n & B \\ \hline C & D \end{array} \right] \begin{bmatrix} e \\ i \end{bmatrix} = MS$$

MNA – General rules

- A branch current is always introduced as an additional variable for a voltage source or an inductor
- For current sources, resistors, conductors and capacitors, the branch current is introduced only if:
 - Any circuit element depends on that branch current
 - That branch current is requested as output

MNA – CCCS and CCVS

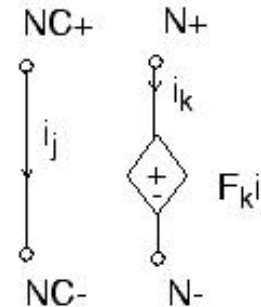
“Stamp”

CCVS:

SPICE: FSK N+ N- NC+ NC- F_K

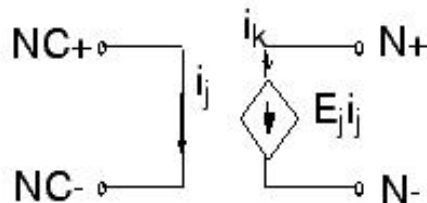
“Stamp”

	N+	N-	NC+	NC-	i_k	i_j
N+					1	
N-					-1	
NC+						1
NC-						-1
branch k	1	-1				$-F_k$
branch j			1	-1		

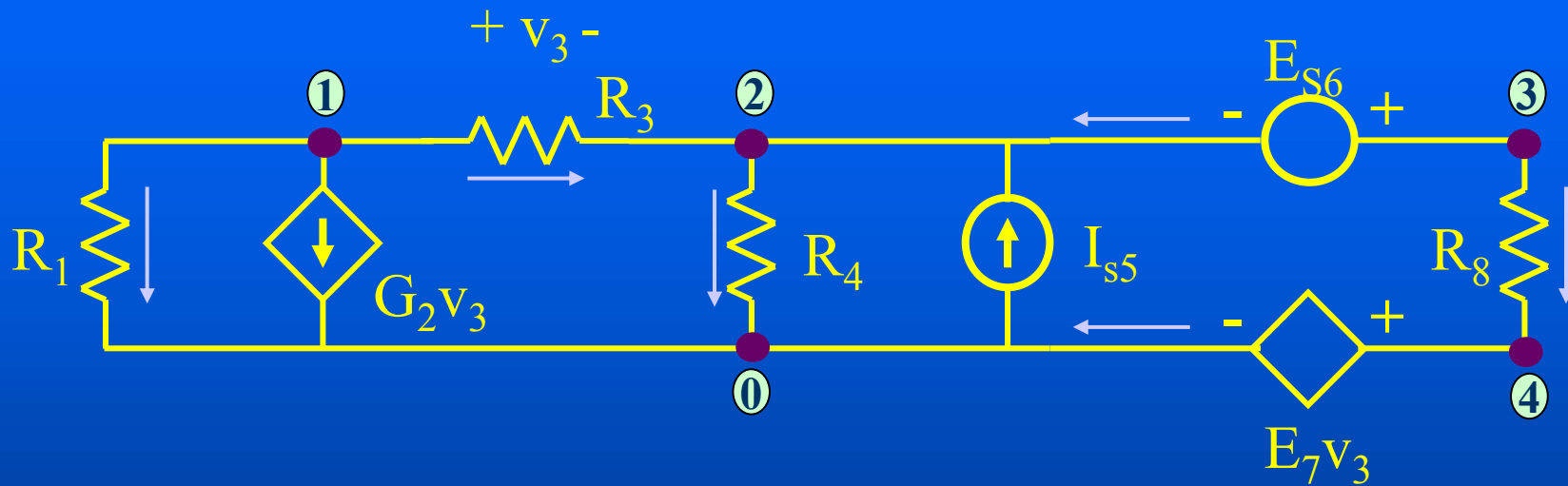


CCCS:

	N+	N-	NC+	NC-	i_j
N+					E_j
N-					$-E_j$
NC+					1
NC-					-1
branch j			1	-1	



MNA – An example



Step 1: Write KCL

$$i_1 + i_2 + i_3 = 0 \quad (1)$$

$$-i_3 + i_4 - i_5 - i_6 = 0 \quad (2)$$

$$i_6 + i_8 = 0 \quad (3)$$

$$i_7 - i_8 = 0 \quad (4)$$

MNA – An example

Step 2: Use branch equations to eliminate as many branch currents as possible

$$1/R_1 \cdot v_1 + G_2 \cdot v_3 + 1/R_3 \cdot v_3 = 0 \quad (1)$$

$$- 1/R_3 \cdot v_3 + 1/R_4 \cdot v_4 - i_6 = i_{s5} \quad (2)$$

$$i_6 + 1/R_8 \cdot v_8 = 0 \quad (3)$$

$$i_7 - 1/R_8 \cdot v_8 = 0 \quad (4)$$

Step 3: Write down unused branch equations

$$v_6 = E_{s6} \quad (b6)$$

$$v_7 - E_7 \cdot v_3 = 0 \quad (b7)$$

MNA – An example

Step 4: Use KVL to eliminate branch voltages from previous equations

$$1/R1 \cdot e1 + G2 \cdot (e1 - e2) + 1/R3 \cdot (e1 - e2) = 0 \quad (1)$$

$$- 1/R3 \cdot (e1 - e2) + 1/R4 \cdot e2 - i6 = is5 \quad (2)$$

$$i6 + 1/R8 \cdot (e3 - e4) = 0 \quad (3)$$

$$i7 - 1/R8 \cdot (e3 - e4) = 0 \quad (4)$$

$$(e3 - e2) = ES6 \quad (b6)$$

$$e4 - E7 \cdot (e1 - e2) = 0 \quad (b7)$$

MNA – An example

$$\left[\begin{array}{cccc|cc} \frac{1}{R_1} + G_2 + \frac{1}{R_3} & -\left(G_2 + \frac{1}{R_3}\right) & 0 & 0 & 0 & 0 \\ -\frac{1}{R_3} & \frac{1}{R_3} + \frac{1}{R_4} & 0 & 0 & -1 & 0 \\ 0 & 0 & \frac{1}{R_8} & -\frac{1}{R_8} & 1 & 0 \\ 0 & 0 & -\frac{1}{R_8} & \frac{1}{R_8} & 0 & 1 \\ \hline 0 & -1 & 1 & 0 & 0 & 0 \\ E7 & -E7 & 0 & -1 & 0 & 0 \end{array} \right] \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ i_6 \\ i_7 \end{bmatrix} = \begin{bmatrix} 0 \\ i_{s5} \\ 0 \\ 0 \\ \hline ES6 \\ 0 \end{bmatrix}$$



$$\left[\begin{array}{c|c} Y_n & B \\ \hline C & 0 \end{array} \right] \begin{bmatrix} e \\ i \end{bmatrix} = MS$$

Modified Nodal Analysis (MNA)

Advantages

- MNA can be applied to any circuit
- Eqns can be assembled directly from input data
- MNA matrix is close to Y_n

Limitations

- Sometimes we have zeros on the main diagonal