

CSE245 Computer Aided Circuit Simulation and Verification

Homework #4. Due 11/9/2004

1. Gilbert/Peierls Factorization

Given a 7x7 lower triangular matrix L

$$L = \begin{bmatrix} 1 & & & & & & \\ x & 1 & & & & & \\ 0 & 0 & 1 & & & & \\ x & x & x & 1 & & & \\ 0 & x & 0 & 0 & 1 & & \\ x & 0 & x & 0 & 0 & 1 & \\ x & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and a 7x1 right hand side vector b

$$b = [0 \quad x \quad x \quad 0 \quad 0 \quad 0 \quad 0]^T$$

We solve $Lx = b$ to obtain vector x, $x = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7]^T$

Questions:

- Construct a directed graph $G(L)$ from the L matrix. The graph $G(L)$ has 7 vertices, it has an edge from vertex j to vertex i if and only if $L_{ij} \neq 0$. Use this graph and the right hand side b to derive the non-zero pattern of vector x.
- In order solve for x_4 , which element(s) of vector x should have been computed before we solve for x_4 ?

2. Moment Matching Property of PRIMA.

To prove the moment matching property of PRIMA, which assumption(s) is(are) necessary?

3. Proof of SOR Convergence

A necessary and sufficient condition for the convergence of SOR method is the magnitude of the largest eigenvalue of the matrix G is smaller than 1. The matrix G is:

$$G = (D - wL)^{-1} (wU + (1 - w)D)$$

Let $Q = w^{-1}(D - wL)$, we have:

$$G = I - Q^{-1}A$$

Let λ be an eigenvalue of G , and let x be an eigenvector corresponding to λ . That is $\lambda x = Gx$. Let $y = (I - G)x$.

Suppose A is positive definite Hermitian.

Questions:

a) prove

$$(w^{-1}D - L)y = Qy = Ax$$

b) prove

$$(w^{-1}D - D + U)y = AGx$$

c) from a) and b) we have

$$\langle (w^{-1}D - L)y, y \rangle = \langle Ax, y \rangle \text{ and,}$$

$$\langle (w^{-1}D - D + U)y, y \rangle = \langle AGx, y \rangle$$

Where $\langle x, y \rangle$ is the inner product of vector x and y , i.e. $\langle x, y \rangle = y^* x$. Here y^* is the conjugate transpose of y .

Prove the eigenvalue λ is less than one when $0 < w < 2$, i.e. SOR converges.

(hint, if matrix A is Hermitian, that is, $A^* = A$, we have $\langle Ax, x \rangle = \langle x, Ax \rangle$. Since A is Hermitian, we also have $L = U^*$)

4. Solution of Equations by Iterative Methods

Consider the linear system

$$\begin{pmatrix} 2 & 0 & -1 \\ -2 & -10 & 0 \\ -1 & -1 & 4 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -12 \\ 2 \end{bmatrix}$$

Using the starting vector $x^{(0)} = (0, 0, 0)^T$, carry out two iterations of each of the following methods:

a. Jacobi b. Gauss-Seidel c. Steepest Decent d. Conjugate Gradient