CSE166 – Image Processing – Midterm

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`http://www-cse.ucsd.edu/classes/fa03/cse166`

3:00-3:50pm Mon. Nov. 3, 2003.

On this exam you are allowed to use a calculator and one 8.5" by 11" sheet of notes. The total number of points possible is 60. In order to get full credit you must **show all your work**. Good luck!

1. (10 pts) Consider the system $g(x) = \frac{1}{4}[f(x-1) - 2f(x) + f(x+1)]$.
   (a) Is this system linear? If it is linear, what is the impulse response?
   (b) Is this system shift invariant?

2. (25 pts) Recall that the even-symmetric Gabor filter has form of a Gaussian times a cosine:

$$h(x) = e^{-x^2/2\sigma^2} \cos(2\pi u_o x)$$

   Assume in this problem that x is continuous.
   (a) Sketch two examples of $h(x)$. Label each drawing to show what σ and u_o control.
   (b) What type of filter is $h(x)$: lowpass, bandpass, or highpass?
   (c) Write down and sketch the Fourier transforms of the following functions (up to a constant scale factor):
   - $e^{-x^2/2\sigma^2}$
   - $\cos(2\pi u_o x)$
   - $h(x)$

   Indicate σ and u_o in each sketch.

3. (10 pts) As discussed in class, histogram equalization works perfectly in the continuous case, but only approximately in the discrete case. Explain why this is true, and give a concrete example.

4. (15 pts) Recall that the chi-squared distance between a pair of histograms $h_i(k)$ and $h_j(k)$ is given by:

$$\chi^2(i,j) = \frac{1}{2}\sum_{k=1}^{K} \frac{[h_i(k) - h_j(k)]^2}{h_i(k) + h_j(k)}$$

   Assume $h_i(k) > 0$ and $h_j(k) > 0$ for all k, and $\sum_k h_i(k) = 1$ and $\sum_k h_j(k) = 1$.
   (a) A simpler distance function one could use is the sum of squared distance (SSD), which is given by $d^2(i,j) = \frac{1}{2}\sum_{k=1}^{K}[h_i(k) - h_j(k)]^2$. What role does the denominator play in the chi-squared definition, and what advantage does this have over SSD?
   (b) What is the chi-squared distance between two identical histograms? Show that $\chi^2(i,j)$ cannot be smaller than this value.
   (c) What is the largest possible chi-squared distance between two histograms? Show that $\chi^2(i,j)$ cannot exceed this value.