CSE166 – Image Processing – Homework #1
Instructor: Prof. Serge Belongie
`http://www-cse.ucsd.edu/~sjb/classes/fa02/cse166`
Due (in class) 3:00pm Wed. Oct. 9, 2002.

Reading

- Skim GW Ch. 1 and 2.

- GW 2.6, 3.5, 3.6.0–3.6.1, 3.7.0–3.7.1

- GW Review Material Ch. 3, "A Brief Overview of Linear Systems," up to the first occurrence of the word "Fourier." (Available at http://www.imageprocessingbook.com.)

Written exercises

1. GW, Problem 3.17.

2. GW, Problem 3.18.

3. GW, Problem 3.22.

4. Determine whether each of the following systems is (i) linear and (ii) spatially invariant. Justify your answers.

   (a) $g(x) = e^{f(x)}$

   (b) $g(x) = f(x)f(x-1)$

   (c) $g(x) = f(x) - f(x-1)$

   (d) $g(x) = f(x-2) - 2f(x-17)$

   (e) $g(x) = [\sin(6x)]f(x)$

   (f) $g(x) = \sum_{k=x-2}^{x+4} f(k)$

   (g) $g(x) = xf(x)$

   (h) $g(x) = f(2x)$

Matlab exercises

1. Working with Images in Matlab.

   (a) Download Figure 1.14(c) from the textbook website photo gallery. One way of doing this is as follows. First, from the Unix/Linux command line, use `wget` to grab the image as follows:

   `wget http://www.imageprocessingbook.com/images/bookimages/images_chapter_01/`
   `Fig1.14\(c\).jpg`

   Note the backslashes before the parentheses. Alternatively, you can get this URL by cutting and pasting it from your browser after locating the image you want in the photo gallery.

   (b) In Matlab, use `imread` to load in the image,

   `I=imread('/home/sjb/images/Fig1.14(c).jpg');`

   where in this case I have assumed that `wget` put the file into `/home/sjb/images/`. Finally, convert the image from `uint8` to `double` by typing `I=double(I)`.

   (c) Familiarize yourself with the following commands: `image`, `imagesc`, `truesize`, `pixval`, `imcrop`, `getrect`, `colormap`, `colorbar`.

(d) Make an m-file script using the above commands to do the following for the image I. Display it as it appears in the figure in the book, using the proper aspect ratio. Interactively select a rectangle roughly enclosing the bottom half of the middle bottle. Crop the image using that rectangle. Display the cropped image using a gray colormap and colorbar, with the maximum and minimum gray levels corresponding to the maximum and minimum brightnesses in the image.

Things to turn in:

- Printout of your m-file.
- Printouts of program output (both the full and cropped images).

2. Moving Averages.

(a) Convolution can be thought of as a moving average. The values of the convolution kernel give the weights on each pixel used in the average. Consider the kernel h=[1 1 1]/3. Create a vector x of length 100 representing a step edge with additive Gaussian noise. Make the height of the edge 1.0 and the standard deviation of the noise 0.05. (Hint: add 0.05*randn(1,100) to x.) Convolve x with h and make plots before and after.

(b) Given y=conv(x,h) with h as given above, write down an expression for the value y(k) for generic k. What is the qualitative effect of convolution with h? This kernel is known as a 3-tap boxcar lowpass filter, or simply a box filter.

(c) Set h equal to the fifth row of Pascal's triangle and normalize it so that it sums to 1. Plot this kernel using stem. This is known as a 5-tap binomial lowpass filter. Perform the convolution with x and produce the plots as before. Compare this result (qualitatively) to the previous result using the boxcar. In particular, why might one want to use one filter vs. the other?

(d) Set x equal to row 375 of Figure 3.35(a). Convolve this signal with each of the above kernels and plot the results.

Things to turn in:

- Printout of plots for part 2a.
- Written answer for part 2b.
- Printout of plots and written answer for part 2c.
- Printout of plots for part 2d.

3. Moving Differences.

(a) When the convolution kernel has negative values, we can think of it as a moving difference. This kind of kernel is useful for edge detection and texture analysis. Consider the kernel [1 0 -1]/2. This is known as the centered first difference. The kernel [1 -1], which is less commonly used, is known simply as a first difference. As in Exercise 2, write down an expression for y(k) for each of these kernels. Why do you suppose the centered version is more commonly used?

(b) Here are two ways of writing the derivative of a function $f'(x)$ as a limit:

$$f'(x) = \lim_{\epsilon \to 0} \frac{f(x + \epsilon) - f(x)}{\epsilon} \quad \text{or} \quad f'(x) = \lim_{\epsilon \to 0} \frac{f(x + \epsilon) - f(x - \epsilon)}{2\epsilon}$$

Explain how these definitions are related to the two first difference kernels. What does ϵ represent when $f(x)$ comes from an image?

(c) Apply the centered first difference kernel to row 238 of Figure 3.35(a) and plot the result, before and after. This time, when you plot the signals, crop the filtered signal by removing its first and last samples. This will make it the same length as the original signal and will also align it. Explain the result: what do the positions, heights, and signs of the spikes represent?

(d) Write down the expression for the centered second difference. Compare it to the limit definition of the second derivative. Explain how to obtain this kernel from the centered first difference kernel.

Things to turn in:

- Written answers for part 3a, 3b, 3d.
- Printout of plots and written answer for part 3c.

4. Sampling and Aliasing.

(a) Consider the function $f(x) = \cos(\omega_o x)$ with $\omega_o = 2\pi k/N$. Let $N = 16$ and consider the interval $x \in [0, 16]$. Let $f(n)$ denote $f(x)$ sampled at the integers $0, 1, \ldots, 16$. In the same graph, plot the continous function $f(x)$ (using `fplot`) and $f(n)$ (using `stem`) for $k = 0, 1, \ldots, 16$. Use `subplot` to arrange the plots in an array on a single sheet of paper. As an illustration, the following plot shows the case for $k = 2$.
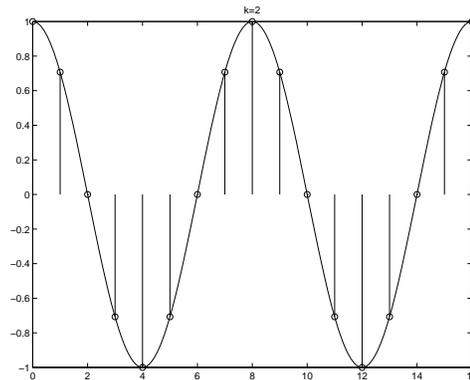
(b) Indicate the value of ω_o at which $f(x)$ hits the Nyquist frequency. Note that at this point we can write $f(n) = (-1)^n$.

(c) Now look up the word "alias" in the dictionary and write down the definition. Explain why this term is used to describe $f(n)$ when ω_o exceeds the Nyquist frequency.

Things to turn in:

- Printout of m-file and plot for part 4a.
- Written answer for parts 4b, 4c.