Computer Science and Engineering
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CSE101
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Order Notation Cheat Sheet

It was brought to my attention that some class members have not seen formal defintions of order notation before. Since I assumed that these had been seen in my review, I have drawn up this cheat sheet on formal definitions. Please note that the definition numbering in this document is my own, and is in no way related to the book or material presented in class.

Definition 1.1 *O-notation*

For a given function $g(n)$, $O(g(n))$ *is defined as:*

$O(g(n)) = \{\ f(n)$ *: there exist positive constants* c *and* n_0 *such that* $0 \leq f(n) \leq c \cdot g(n)$, *for all* $n \geq n_0\}$

When we say $f(n) = O(g(n))$, or $f(n)$ is $O(g(n))$, what we actually mean is that $f(n)$ is an element of the set $O(g(n))$. **This is true for all order notation.**

Intuitively, this means that $g(n)$ is an upper bound on $f(n)$ (analogous to $g(n) \geq f(n)$ for real numbers).

For example, if $f(n) = n^3 + 4n^2 + 5$, then we can say $f(n) = O(n^4)$, or $f(n)$ is $O(n^4)$. This is true because $f(n) \leq cg(n)$, for $c = 10$, $n_0 = 1$. Also, note that $f(n) = O(n^3)$.

Definition 1.2 *o-notation*

For a given function $g(n)$, $o(g(n))$ *is defined as:*

$o(g(n)) = \{\ f(n)$ *: for any positive constant* c, *there exists a constant* n_0 *such that* $0 \leq f(n) < cg(n)$, *for all* $n \geq n_0\}$

Intuitively, this means that $g(n)$ is an upper bound of $f(n)$ that is not tight (analogous to $g(n) > f(n)$ for real numbers).

For example, if $f(n) = n^3 + 4n^2 + 5$, then we can say $f(n) = o(n^4)$ or f(n) is $o(n^4)$. Note that $f(n)$ is **not** $o(n^3)$.

Definition 1.3 Θ*-notation*

For a given function $g(n)$, $\Theta(g(n))$ is defined as:

$\Theta(g(n)) = \{$ *$f(n)$: there exist positive constants c_1, c_2, and n_0 such that* $0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$, *for all* $n \geq n_0\}$

Intuitively, this means that $g(n)$ is roughly equivalent to $f(n)$ (analogous to $g(n) \approx f(n)$ for real numbers).

For example, if $f(n) = n^3 + 4n^2 + 5$, then we can say $f(n) = \Theta(n^3)$ or $f(n)$ is $\Theta(n^3)$.

Definition 1.4 *Ω-notation*

For a given function $g(n)$, $\Omega(g(n))$ is defined as:

$\Omega(g(n)) = \{$ *$f(n)$: there exist positive constants c_1 and n_0 such that* $0 \leq c_1 g(n) \leq f(n)$, *for all* $n \geq n_0\}$

Intuitively, this means that $g(n)$ is a lower bound on $f(n)$ (analogous to $g(n) \leq f(n)$ for real numbers).

For example, if $f(n) = n^3 + 4n^2 + 5$, then we can say $f(n) = \Omega(n^3)$ or $f(n)$ is $\Omega(n^3)$. Also, we have that $f(n) = \Omega(n^2)$.

Definition 1.5 *ω-notation*

For a given function $g(n)$, $\omega(g(n))$ is defined as:

$\omega(g(n)) = \{$ *$f(n)$: for any positive constant c, there exists a constant n_0 such that* $0 \leq cg(n) < f(n)$, *for all* $n \geq n_0\}$

Intuitively, this means that $g(n)$ is lower bound of $f(n)$ that is not tight (analogous to $g(n) < f(n)$ for real numbers).

For example, if $f(n) = n^3 + 4n^2 + 5$, then we can say $f(n) = \omega(n^2)$ or $f(n)$ is $\omega(n^2)$. However, $f(n)$ is **not** $\omega(n^3)$.