

## CSE105 A Solutions to Quiz 3, Tuesday Nov. 27, 2001

- [5] 1. State the Church-Turing Thesis:

Intuitive notion of algorithms = Turing machine algorithms

- [8] 2. Fill in the blanks in the following proof.

**Theorem** If  $A \leq_m B$  and  $B$  is Turing-recognizable, then  $A$  is Turing recognizable.

**Proof.** Let  $M$  be the recognizer for  $B$ , and let  $f$  be the reduction from  $A$  to  $B$ . That is,  $f : \Sigma^* \rightarrow \Sigma^*$  is a computable function such that, for every  $w \in \Sigma^*$ ,

$$w \in A \Leftrightarrow \underline{f(w) \in B}$$

We define the recognizer  $N$  for  $A$  as follows:

“On input  $w$ :

- (a) Compute  $x = \underline{f(w)}$ .
- (b) Run  $M$  on  $x$  and output whatever  $M$  outputs.”

$N$  recognizes  $A$  because  $M$  is a recognizer for  $B$ .

- [7] 3. Argue that the class of decidable languages is closed under complementation, by filling in the blanks in the following proof.

Let  $A \subseteq \Sigma^*$  be an arbitrary decidable language, and let  $N$  be a TM deciding  $A$ . That is, for every  $w \in \Sigma^*$ , the TM  $N$  on  $w$  outputs Accept if  $w \in A$ , and outputs Reject otherwise.

Construct a decider  $M$  for the complement  $\bar{A}$  of  $A$  as follows:

“On input  $w$ :

- (a) Run  $N$  on  $w$ .
- (b) If  $N$  accepts  $w$ , then reject; otherwise accept.

$M$  halts on every input  $w$  because  $N$  is a decider for  $A$ .

- [3] 4. Give an example of a language that is *not* Turing-recognizable, *but* its complement is Turing-recognizable. (Just describe the language; you do not need to give any proofs.)

$\bar{A}_{\text{TM}}$ , (i.e., the complement of  $A_{\text{TM}}$ ).

- [10] 5. Show that the language  $A_{\text{TM}} = \{\langle M, w \rangle \mid M \text{ is a TM and } M \text{ accepts } w\}$  is undecidable by filling in the blanks in the following proof.

Define a language  $D$  so that  $D \neq L(N)$ , for every TM  $N$ . Namely, for every string  $\langle N \rangle$ , where  $N$  is a TM, we define

$$\langle N \rangle \in D \Leftrightarrow N \text{ on input } \langle N \rangle \text{ does not accept}. \quad \square$$

For an arbitrary TM  $N$ , we have  $L(N) \neq D$  because the string  $\langle N \rangle$  is in one of the languages  $L(N)$  and  $D$ , but not in the other.

Now, if  $A_{\text{TM}}$  is decidable by some TM  $H$ , then we obtain a decider  $K$  for the language  $D$  as follows:

“On input  $\langle N \rangle$ , where  $N$  is a TM:

- (a) Run  $H$  on input  $\langle N, \langle N \rangle \rangle$ .
- (b) If  $H$  accepts, then reject. Otherwise accept”

Thus,  $D = L(K)$ , which is a contradiction. Hence,  $A_{\text{TM}}$  is undecidable.

[12] 6. For each of the following statements, indicate if it is **True** or **False**. (No justification is necessary.)

- (a)  $A_{\text{TM}}$  is Turing-recognizable.      **True**
- (b)  $A_{\text{DFA}}$  is undecidable.      **False**
- (c)  $A_{\text{CFG}}$  is undecidable.      **False**
- (d) Every CFL is decidable.      **True**
- (e) Every language accepted by a 2-stack PDA is decidable.      **False**
- (f) The complement of  $\text{HALT}_{\text{TM}}$  is Turing-recognizable.      **False**