

CSE105 A Closed Book Quiz 2, Tuesday Nov. 6, 2001

Time Limit: 40 minutes

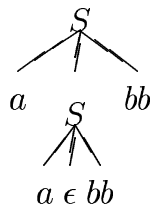
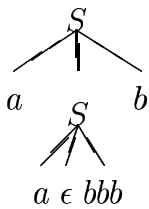
NAME	1	2	3	4	5	Total (50)
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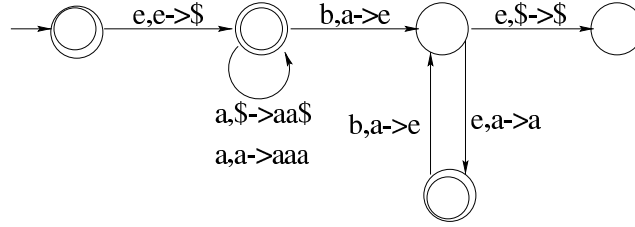
- [10] 1. (a) Give a CFG generating the language $L = \{a^n b^m \mid 0 \leq n \leq m \leq 3n\}$.

$$S \rightarrow aSb \mid aSbb \mid aSbbb \mid \epsilon$$

- [5] (b) Is your grammar ambiguous? Why or why not?

Yes, it is ambiguous. There are two different parse trees for deriving the same string $aabbbb$:





- [10] 2. Give the state diagram of a PDA that recognizes the language

$$M = \{a^i b^j \mid i, j \geq 0 \text{ and } j < 2i\}$$

Note: your PDA is allowed to push more than one symbol onto the stack at a time; if it pushes a string $a_1 a_2 \dots a_k$, then the symbol a_1 will become the top of the stack.

Idea: for each input symbol a , push two a 's onto the stack. When see the first b , start popping a 's. Accept while the stack remains nonempty. Reject if the stack becomes empty.

- [5] 3. (a) Complete the definition: A CFG $G = (V, \Sigma, R, S)$ is in *Chomsky Normal Form* if each rule in R is of the form

$$A \rightarrow \underline{BC}$$

or

$$A \rightarrow \underline{a},$$

where $A \in V$, $B, C \in V - \{S\}$, $a \in \Sigma$.

In addition, we allow the rule $S \rightarrow \epsilon$.

- [5] (b) Convert the following CFG into an equivalent CFG in Chomsky Normal Form. (The start variable is A .)

$$A \rightarrow BAB \mid B \mid 000$$

$$B \rightarrow 00 \mid AB$$

Solution:

$$S \rightarrow BX \mid ZZ \mid AB \mid ZY$$

$$A \rightarrow BX \mid ZZ \mid AB \mid ZY$$

$$B \rightarrow ZZ \mid AB$$

$$X \rightarrow AB$$

$$Y \rightarrow ZZ$$

$$Z \rightarrow 0$$

The new start variable is S .

- [10] 4. Use the Pumping Lemma to show that the following language is *not* context-free:

$$L = \{w \mid w \in \{a, b, c\}^* \text{ and } w \text{ has equal number of } a\text{'s, } b\text{'s, and } c\text{'s}\}$$

Suppose L is a CFL. Then the Pumping Lemma holds for L with some pumping length p . Consider the string $s = a^p b^p c^p$. Clearly, $s \in L$ and $|s| > p$. By the Pumping Lemma, there must be a partitioning of $s = uvxyz$ such that $|vy| > 0$, $|vxy| \leq p$, and $uv^i xy^i z \in L$ for every $i \geq 0$. Consider all possible cases: Since $|vxy| \leq p$, v and y contain at most two different (consecutive) symbols: type 1 and type 2. Hence, if we take the string $uvvxyyz$, then it will contain more symbols of type 1 or of type 2, while it will still contain the same number of symbols of the remaining type (type 3). Hence, $uvvxyyz \notin L$. A contradiction.

- [5] 5. Complete the following high-level description of a Turing machine recognizing the language

$$L = \{w \mid w \in \{a, b, c\}^* \text{ and } w \text{ has equal number of } a\text{'s, } b\text{'s, and } c\text{'s}\}$$

On input w :

- (a) If $w = \epsilon$, then Accept.
- (b) Scan the input and find the leftmost a . Replace it by X . If no a is found, Reject.
- (c) Scan the input and find the leftmost b . Replace it by Y . If no b is found, Reject.
- (d) Scan the input and find the leftmost c . Replace it by Z . If no c is found, Reject.
- (e) Scan the input and check if any a 's, b 's, or c 's are left. If none, then Accept
Otherwise go to Step (b)