Joint Time Synchronization and Tracking for Mobile Underwater Systems

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ABSTRACT
Time synchronization and localization are key requirements for distributed underwater systems consisting of numerous low-cost submersibles. In these systems, submersibles are highly resource constrained and typically have limited acoustic communication capability. We investigate the problem of time synchronization and tracking for submersibles that only have the capability to receive acoustic signals. Traditional Long Base Line (LBL) systems track the location of submersibles by providing a GPS-like infrastructure that consists of a few reference beacons at known locations. In these systems the unknown positions of submersibles are estimated from beacon transmissions using time-difference-of-arrival (TDoA) based localization. As such TDoA makes the key assumption that beacon transmissions occur nearly concurrently in time. While this assumption is ensured in small LBL deployments it does not hold as the size of the system scales up. In this paper we identify scenarios where signals from multiple beacons are significantly lagged in time. We further identify the motion of the submersible between signal arrivals as a key factor that deteriorates the performance of TDoA, when transmissions are not concurrent. To address this problem we propose to track the submersible while performing time-synchronization. Our proposed technique, called Time of Arrival based Tracked Synchronization (ToA-TS) essentially extends GPS like localization for scenarios where beacon transmissions are not concurrent and submersibles are not capable of two-way communication.

Categories and Subject Descriptors
C.2.3 [Computer-Communication Networks]: Network Operations; C.4 [Performance of Systems]: [Design studies]

General Terms
Algorithms, Performance.

Keywords
Underwater Networks, Real-time Tracking, Acoustic Networks

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1. INTRODUCTION
Mobile underwater submersibles are widely used in many oceanographic applications. As submersibles become more agile, they present new opportunities to study oceanic phenomena that are dynamic and vary at smaller spatial scales (tens to hundreds of meters). To aid such exploration there has been a growing interest to deploy numerous low-cost and compact submersibles as swarms that operate in a distributed and coordinated fashion. A crucial feature of such distributed systems is that the cost and capabilities of individual submersibles are scaled down in order to scale up the spatial extent and density of the sampling system. As a result, submersibles are highly energy constrained, and typically have very short-range acoustic communication or in many cases only have the ability to act as passive receivers.

Determining the location and time of submersibles underwater in a global frame of reference are two key requirements of distributed systems. Global position and time can be easily obtained from GPS for terrestrial systems. However, due to the unavailability of GPS underwater, traditional underwater acoustic navigation techniques typically rely on a GPS-like infrastructure to track submersibles. This infrastructure consists of a few reference beacons (usually four) that are deployed in precise configurations as in Long Baseline Systems (LBL) [1]. The beacons essentially act as 'GPS satellites' for underwater by deducing their own position and time from GPS. The submersible's position and local time are jointly estimated from measurements of the time-difference-of-arrival (TDoA) of acoustic signals that are transmitted by the beacons[1]. TDoA is especially relevant to tracking and synchronizing low-cost submersibles that have limited acoustic communication capability and can seldom transmit to surface beacons. However, TDoA makes two crucial assumptions during the interval that the time-of-arrival measurements are obtained (a) the submersible is stationary, and (b) the difference between the global time according to beacons and the local time of the submersible, (i.e., clock offset) remains constant. In general, the position of the submersible changes due to its motion and the clock offset changes at an unknown rate, which is known as the clock drift. Therefore, while the assumptions made by TDoA do not hold in general, they are satisfied in practical LBL systems by ensuring that the submersible receives four signals in succession over a short time window as depicted in Figure 1(a). In such a scenario, the movement of the submersible is negligible during the

[1] Other acoustic navigation techniques are also possible within this basic framework. We discuss these techniques in related work (Section 7). However, these approaches typically rely on bi-directional communication with beacons. Therefore, they are not suitable for tracking submersibles with only receive capability.

signaling period. Similarly, its clock offset is virtually constant (i.e., the effect of drift is negligible).

The problem that we address in this paper is time synchronization and tracking submersibles when signals from multiple beacons no longer arrive concurrently, but rather arrive over a much longer time epoch, *T* as depicted in Figure 1(b). We will explain shortly why this problem arises in the context of swarm systems. The key point is that when signals no longer arrive concurrently at each submersible, two effects come into play (that are ignored by TDoA based localization): the submersible can significantly move between signal arrivals and the clock offset may not remain constant over the signaling period due to the cumulative effect of clock drift. In this paper we identify scenarios where the error introduced due to these two factors becomes significant for a nominal range of relevant parameters (a detailed discussion is presented in Section 3). When there is significant motion between signal arrivals, in order to time synchronize the submersible, it has to be additionally tracked over the time epoch *T*. Therefore, the key problem that we will solve is joint time synchronization and tracking of submersibles based *only* on one-way acoustic transmissions from beacons. In the next section we delineate why this problem arises in the context of swarm systems and discuss our solution strategy. Our proposed technique, called Time of Arrival based Tracked Synchronization (ToA-TS) extends GPS like localization for scenarios where beacon transmissions are not concurrent and submersibles are not capable of two-way communication.

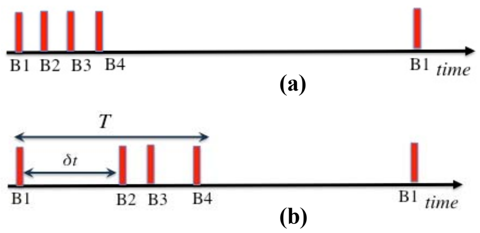


Figure 1: Signals received at a submersible from four beacons in (a) LBL systems and (b) Swarm systems

2. PROBLEM MOTIVATION

In scenarios where beacon transmissions are spread over a time window *T,* two key effects come into play. However, prior to discussing these effects in more detail, in this section we discuss why signals from transmissions are likely to occur over longer time epochs in swarm systems compared to traditional LBL deployments.

Traditional LBL systems are designed for small deployments typically consisting of a few beacons (typically four), whose acoustic transmissions can be received by all submersibles within the deployment. As such LBL provides sufficient localization accuracy for such deployments. This is because signals from few beacons can be scheduled to occur within a short time window. Since all the submersibles are within the communication range of the beacons, these signals also arrive at near concurrent times at each submersible as shown in Figure 1(a). However, as the spatial extent and size of the system is scaled up, this scenario changes.

As the spatial extent of the deployment increases, more beacons are required to provide sufficient acoustic coverage for localizing submersibles. Further, beacons may not be deployed in precise configurations. To avoid collisions, transmissions from multiple beacons have to be sufficiently lagged over a time epoch, *T*. Given that propagation delays are in the order of seconds, the time lag between beacon transmissions is likely to be in the order of tens of seconds or more. As a result transmissions from multiple beacons can no longer be scheduled to occur concurrently. The crucial point here is that while we still propose a system where all beacons try to schedule their transmission as close as possible in time, these transmissions cannot be concurrent due to the size of the network. A submersible within this network can only receive signals from a subset of beacons. Further, this subset changes due to the motion of the submersible and possibly that of the beacons [9]. Therefore, the time lag, δt between transmissions from a random subset as received by a submersible can be significant as depicted in Figure 1(b).

In such scenarios two additional factors come into play, which were previously ignored by TDoA, namely, the motion of the submersible and the drift in its local clock between signal arrivals. In the next section we discuss scenarios where these two factors can significantly affect the performance of TDoA, which assumes concurrent transmissions.

3. TDoA LOCALIZATION WITH NON-CONCURRENT SIGNAL ARRIVALS

First, we briefly describe the time-difference-of-arrival (TDoA) method, which is typically used in LBL systems. In the subsections that follow we will consider the individual effects of submersible motion and clock drift on the performance of TDoA, when acoustic signals from beacons do not occur concurrently.

In TDoA based localization the difference in the measured time of arrivals of signals received from a pair of beacons translates to the difference in range estimates with those beacons. As a result each estimate of time-difference gives rise to a hyperbola for the unknown position of the submersible. A unique estimate of the submersible's position can be obtained by intersecting three such hyperbolas (obtained from four beacon transmissions). However, this technique requires that beacons transmit at near concurrent times. For the scenario where transmissions are lagged, this approach essentially neglects the motion and clock drift of the submersible between signal arrivals. In reality, both these factors add an error to the measured time differences. As a result, each hyperbola thus obtained does not represent the submersible's position at any particular time. Therefore, the hyperbolas no longer intersect at a common point. A least squares approach can still be used to obtain an estimate for the position. However, these errors are likely to degrade the performance of TDoA based localization.

In the next subsections we identify scenarios where the error due to motion and/or the clock drift become significant. We consider errors greater than a meter to be significant because errors due to noise, surface reflections and refraction and even hardware are expected to be of that order [2][3]. For this analysis we denote the lag between a pair of signals as δt, the submersible's speed as v, the clock drift as η, and the speed of sound as c.

3.1 Effect of motion

TDoA assumes that the difference in signal arrivals correspond to difference in range estimates with beacons at a particular time instance. As a result, when the submersible is moving, the error in distance (more specifically the error in the difference in distance estimates with two beacons) is given by the displacement of the submersible in the signaling interval $v\delta t$. We evaluated this error as a function of the signaling interval δt for nominal submersible speeds of 0.4 m/s to 2 m/s.

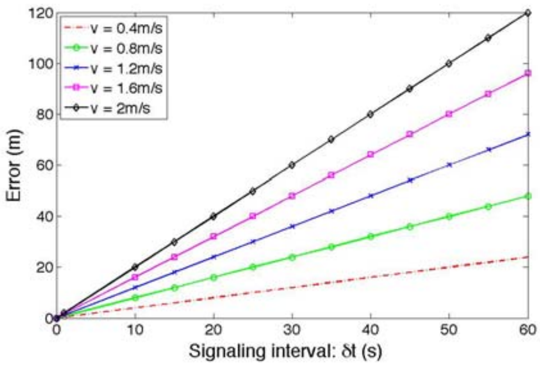


Figure 2: Error in distance due to the submersible's motion

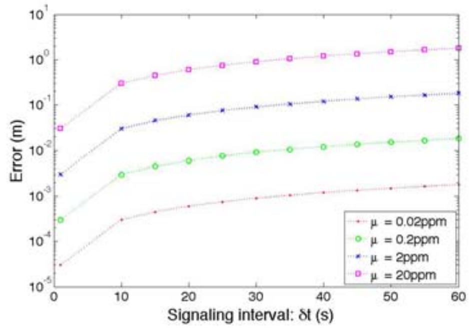


Figure 3: Error in distance due to the clock drift

As shown in Figure 2, the error due to the submersible's motion becomes significant when the signaling interval, δt, increases to more than a few seconds. Further, this error can be as large as 120 m when the signaling interval is one minute long. Therefore, the effect of the submersible's motion can significantly degrade the performance of TDoA when transmissions are non-concurrent.

3.2 Effect of clock drift

Next, we consider the effect of a changing clock offset. The change in the clock offset during the signaling interval essentially translates to an error in the measured time-difference between the signal arrivals from a pair of beacons. Since the clock offset varies linearly with the drift, the change in the clock offset during the signaling interval is computed as $\eta\delta t$. When multiplied by the speed of sound, this translates to an error in (the difference in) range estimates with the beacons, given by $c\ \eta\delta t$. This error is shown as a function of the signaling interval in Figure 3 for clock drifts in the range of 0.02 ppm to 20 ppm. As the figure shows the error due to clock drift is not significant when the drift is less than 20 ppm.

Based on the above results we propose that submersibles with only receive capability incorporate low-drift clocks, especially if they are to operate in scenarios where beacon transmissions are non-concurrent. Consequently, the clock drift does not have to be estimated over short time epochs, which makes the estimation problem more tractable. Low-drift clocks have been incorporated in a number of practical underwater platforms. For example, Eustice et al have reported integrating a temperature clock into the micro-modem, with a skew of only 0.02 ppm [4]. Similar levels of accuracy have also been achieved through a novel and inexpensive approach using two crystals [5].

While the effect of clock drift is negligible when an accurate clock is used, the effect of the motion can still be significant. Although the error introduced due to motion cannot be eliminated entirely, we propose to compensate for motion as much as possible by adding of an IMU (Inertial Measurement Unit) and further tracking the submersible's position during the window, T. As such, it is not atypical for submersibles to have an on-board IMU for tracking their location underwater. However, TDoA does not use information from an IMU, per se.

Based on the above reasoning, we consider the use of both an on-board IMU as well as a low-drift clock crystal to jointly track and synchronize a submersible. Next, we present our proposed Time of Arrival based Tracked Synchronization (ToA-TS) technique. Our goal is to extend GPS like localization for systems where beacon transmissions do not occur concurrently and submersibles are not capable of two-way communication.

4. PROBLEM FORMULATION

We consider the problem of tracking a submersible using acoustic messages that are transmitted at different times by multiple reference beacons over a period T. The beacons deduce their location and time from GPS. However, the submersible is not time-synchronized with the beacons. Our goal is to estimate the Maximum Likelihood (ML) position and clock offset of the submersible over a time epoch T given the send and receive timestamps of acoustic messages transmitted by the beacons and IMU measurements obtained during that time. Next, we present a mathematical formulation of our tracking problem.

Each acoustic message transmitted by a beacon includes the position of the beacon as obtained from GPS and a time stamp indicating the time it was sent according to global time. The receive time of the message is recorded according to the local clock of the submersible. The difference in these two times relates the unknown position of the submersible and its clock offset, as given by equation (1).

$$\|P_i(t_k) - P(t_k)\|_2/c = d_i(t_k)/c = T_{k,i} - t_k + \Delta(t_k) + \varepsilon_k \quad (1)$$

where $P_i(t_k)$ and $P(t_k)$ denote the positions of beacon i and the submersible at the time of message transmission, respectively. The transmit time of the k^{th} message according to global time is denoted by t_k. The receive time of the message from beacon i according to the submersible's local time is denoted by $T_{k,i}$. We refer to $T_{k,i} - t_k$ as the *measured* time-of-flight of the acoustic message and denote it as $m_i(t_k)$. $\Delta(t_k)$ is the unknown clock offset of the submersible at the time of message transmission. ε_k is the error in detecting the first arrival of the signal. The statistics of this error is known from previous experimental characterizations performed in prior work [2][3]. We model the statistics as a zero mean Gaussian distribution with standard deviation σ_R.

Acoustic messages are received from multiple beacons over a time epoch, T as depicted in Figure 1(b). Note that TDoA based localization assumes that both the position of the submersible and the clock offset remain constant during this time. From equation (1), we observe that if this were true, the number of unknowns does not increase with each new time-of-flight measurement. The resulting problem is then solvable as long as measurements are made with at least four beacons. We compare our proposed solution with this approach in Section 6.

As discussed earlier, contrary to the assumption made by TDoA, the submersible's motion may be significant during the signaling period. Therefore, we take this motion into account by using measurements of the submersible's velocity that are obtained from an on-board IMU. These measurements relate the position of the submersible at consecutive time steps as given by equation (2).

$$\tilde{v}_k = \left(P(t_{k+1}) - P(t_k)\right)/(t_{k+1} - t_k) + \varepsilon_{v,k} \quad (2)$$

where $\varepsilon_{v,k}$ is the error in velocity measurement. The problem that we address is to track the unknown position and clock offset of the submersible using measurements obtained in intervals of duration *T.* As depicted in Figure 1(b), this interval is chosen as the minimum time in which acoustic messages are received from at least four unique beacons. Since our solution is aimed for submersibles with low-drift clocks, the effect of the clock drift is negligible as discussed earlier in this section. Therefore, we propose to estimate the Maximum Likelihood (ML) positions of the submersible and a single unknown clock offset during the interval *T*. This process is periodically repeated to track and synchronize the submersible over longer durations. In the next section we describe our proposed solution.

5. SOLUTION STRATEGY

Our goal is to estimate the Maximum Likelihood (ML) position and clock offset of the submersible over a time epoch *T* given the *measured* time of flight of acoustic messages transmitted by the beacons and IMU measurements obtained during that time. We assume that the depth is known from pressure sensors, therefore, we estimate the position of the submersible in two dimensions.

The timestamps obtained from acoustic messages impose a non-linear and non-Gaussian constraint on the pdf of the position and clock offset of the submersible at each point in time, as given in equation (1). To incorporate such type of constraints, we appropriately define the tracking problem in the Bayesian estimation framework of factor-graphs. Factor graphs offer a way to represent any global function (in this case the joint probability distribution of the unknown position and clock offset) in terms of simpler local functions that depend only on a subset of variables. The sum-product algorithm can operate on this graph and exploit these simple relations to estimate the pdf of individual states in the graph via iterative message passing [6][7]. We have previously discussed this framework in the context of underwater tracking [8]. However, in our previous work we used this framework to track submersibles from inter-vehicle measurements of *distance*. Distance estimation and time synchronization were performed independent of tracking. Here we use the framework of factor-graphs to jointly track and time-synchronize a submersible. Next, we present the factor-graph description of our tracking problem.

5.1 Factor-graph description

As a first step towards solving our joint tracking and synchronization problem, we have come up with the appropriate factor-graph description given in Figure 4. The graph gives a description of the interdependencies between the unknown positions and clock offset of the submersible over time. Mathematically, this graph describes the joint distribution of the unknowns given all measurements of time-of-flight (prior to synchronization) and measurements of the submersible's motion in the interval *T*. Each state *P*(*t*) in the factor graph (denoted by a circle) represents the unknown position of a submersible at a time instance *t*. The unknown clock offset is denoted as $\Delta(t)$ and represented by a hexagon. The square blocks, known as function-nodes, link the state-variables. The function-nodes not only indicate which state-variables are related but also how they relate. More specifically, a function node *f* that has links to state-variables X and Y assigns a weight *f(*X= *x,* Y= y | *m)* to any outcome X = *x*, Y = *y* given a measurement *m.* Therefore, function-nodes define constraints (or interdependencies) between state-variables. Two types of constraints are captured by the graph. Function-nodes of type f_1 define constraints on the position and clock offset at each point in time as imposed by the *measured* time-of-flight of acoustic messages. Function-nodes of type f_2 describe how the unknown positions vary over consecutive time steps given measurements of the submersible's velocity. The clock offset remains constant over the time epoch *T*, therefore, it is represented by a single unknown state.

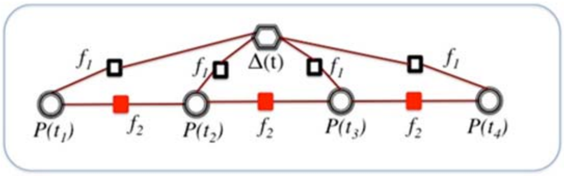


Figure 4: Factor-graph description for joint synchronization and tracking

The key advantage of the factor-graph framework is that function-nodes can take any generic form that best describes the likelihood of measurements given state-variables. This makes it possible to easily describe non-linear relations, such as the one given by equation (1). For our specific problem, the formal definitions of the function-nodes are given in equations (3) and (4). These are derived from equations (1) and (2), respectively.

$$\begin{aligned} f_1(X(t) &= [P(t),\Delta(t)]) \\ &= p\big(P(t),\Delta(t)\big|m_i(t),P_i(t)\big) \\ &= \frac{1}{\sqrt{2\pi}\sigma_R} e^{-(\|P_i(t)-P(t)\|/c-(m_i(t)+\Delta(t)))^2/2\sigma_R^2} \end{aligned} \quad (3)$$

$$\begin{aligned} f_2\big(P(t_k),P(t_{k+1})\big) &= p\big(P(t_k),P(t_{k+1})\big|\overline{v_k}\big) \\ &= \frac{1}{\sqrt{2\pi}\sigma_v} e^{-\|P(t_{k+1})-(P(t_k)+\overline{v_k}\cdot(t_{k+1}-t_k))\|^2/2\sigma_v^2} \end{aligned} \quad (4)$$

As depicted in Figure 4, the resultant factor-graph representation of this problem is cyclic. This poses a key challenge for the tracking algorithm that runs on the graph to estimate the unknown states. This algorithm, which we will later describe in Section 5.2, is not guaranteed to converge for cyclic graphs. To address this problem we use a result by Kschischang et. al. to transform a cyclic factor-graph to an equivalent graph that is free of cycles (Section VI. B[6]). Specifically we apply a "stretching" transformation to the factor-graph in Figure 4, to obtain an equivalent acyclic graph that is shown in Figure 5. This transformation entails augmenting each of the unknown position states *P*(*t*)*,* described in 2D, to include the unknown clock offset, $\Delta(t)$. In effect, $\Delta(t)$ is *stretched* along the path to each unknown position state. The augmented state is denoted as *X*(*t*)=[*P*(*t*), $\Delta(t)$], whose state-space is naturally defined over 3D. As a final step, we have to modify the function-nodes f_2 (in the original graph) to a new function-node, g_2 which takes as input the augmented state *X*(*t*). The function node g_2 is f_2 predicated by the fact that the value of the clock offset does not change over time. g_2 is defined in equation (5).

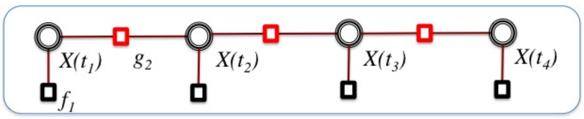


Figure 5: Acyclic factor graph obtained from stretching the clock offset variable to all the other states

$$\begin{aligned} g_2\big(X(t_k),X(t_{k+1})\big) &= p\big(X(t_k),X(t_{k+1})\big|\overline{v_k}\big) \\ &= f_2\big(P(t_k),P(t_{k+1})\big)\cdot I\big(\Delta(t_{k+1}),\Delta(t_{k+1})\big) \end{aligned} \quad (5)$$

where, I is a predicate function given by:

$$I(u,v) = \begin{cases} 1, & if\ u = v \\ 0, & if\ u \neq v \end{cases}$$

The sum-product algorithm runs on the above-described factor-graph to estimate the pdf of the unknown state-variables. The algorithm in its generic form is discussed in previous tutorials [6][7]. Here we present a brief overview and examine its operation in the context of our joint tracking and synchronization problem.

5.2 Tracking Algorithm

Once the factor graph is defined, an iterative message passing algorithm, namely, the sum-product algorithm runs on it to solve the estimation problem. In each iteration of the algorithm, nodes in the factor-graph compose messages that are sent over the links of the graph. Messages are composed based on those received in a previous iteration.

There are two main types of messages exchanged during a single iteration of the algorithm. Messages from a function node *f* to a state-variable *x*, denoted as $\mu_{x\text{-}f}$ (.) and messages from a state-variable x to a function node *f*, denoted as $\mu_{f\text{-}x}$(.). These are computed as per equations (6) and (7) [6]. Messages that are sent out by a state-variable to its neighbor function-nodes are the most current estimate of the state-variable's probability distribution. A message sent out by a function node to its neighbor state-variable is an estimate of the state-variable's probability distribution, given the probability distribution of all other state-variables that are neighbors of the function node. A function node generates its message to a neighbor state-variable by performing a marginalization of its local likelihood function (as described by equation (6)). A state-variable estimates its distribution (or outgoing messages) by intersecting the individual estimates of its distribution provided by its function-node neighbors, (as given by equation (7)) . These messages, computed locally at each node, set up the machinery to carry information across the graph.

$$\mu_{f-x}(x) = \sum_{\sim\{x\}} f(X) \prod_{y \in n(f)\setminus\{x\}} \mu_{y-f}(y) \tag{6}$$

$$\mu_{x-f}(x) = \prod_{h \in n(x)\setminus\{f\}} \mu_{h-x}(x) \tag{7}$$

Where the summary operation is defined in [6].

To better understand how information obtained from acoustic messages and IMU measurements are combined to solve our specific estimation problem, we present the operation of the sum-product algorithm on the factor-graph described in the previous section. We specifically consider messages passed over three iterations of the algorithm on the sub-graph shown in Figure 6.

Iteration1: In the first iteration, information flows from function-nodes of type f_1 to the unknown state-variables, $X(t_1)$ and $X(t_2)$ as shown in Figure 6(1) . Each of these function-nodes is only linked to one state-variable. Therefore, (from equation (6), the message sent from f_1 to its neighbor state-variable, *X*(*t*) is f_1(X(t)), evaluated over the 3D state-space of *X(t)*. This message is an estimate of the pdf of *X(t)*. Its state-space is visualized in Figure 7(a). The x-y axes represent the possible positions of the submersible at time *t*, while the z-axis represents the possible values of the clock offset. The cone shown in the figure highlights the possible position and clock offset values considered jointly, which follows directly from equation (1). As per this equation, for each value of the clock offset the position of the submersible is a ring in 2D centered at the position of the beacon. The radius of the ring is the sum of the measured time of flight and the clock offset. The minimum possible radius is zero since the true time-of-flight (or distance) is always positive. As the value of the possible clock offset increases, so does the radius of the ring. Therefore, we obtain a cone in 3D for the state-space of *X*(*t*) as shown in Figure 7.

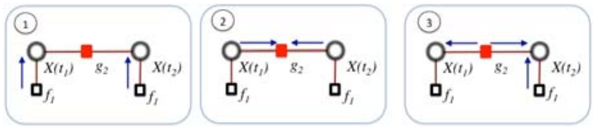


Figure 6: Operation of the sum-product algorithm on a sub-graph

Following equation (3), the message passed by f_1 is a set of weights computed for each point on the cone. This 3D message can be visualized as a number of pdfs in 2D stacked on top of each other. Each 2D pdf is defined for a particular value of the clock offset by intersecting a plane with the cone as shown in Figure 7(a), and computing a set of weights over the ring thus obtained. The pdf computed over the ring by f_1 is visualized in Figure 7(b) and computed as per equation (3) for a fixed value of the clock offset. Therefore, each measurement of time-of-flight obtained from an acoustic message results in the above-described *weighted-cone* as the pdf of the unknown position and clock offset at that time.

Iteration 2: In this iteration, the estimates of the pdf of $X(t_1)$ and $X(t_2)$, from the previous iteration are communicated to the function-node g_2.

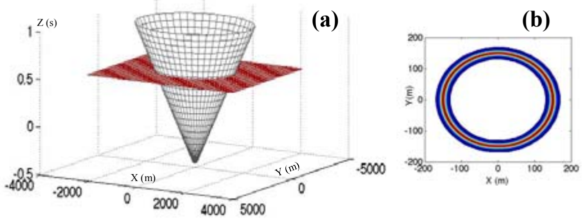


Figure 7: (a) 3D pdf of position and clock offset given the measured time of flight of an acoustic message (b) Cross-section of the pdf in 2D for a fixed value of clock offset

Iteration 3: Function-node g_2 combines these messages to estimate the pdf of each of its neighbor state-variables. The message passed from g_2 to the state-variable $X(t_2)$ is an estimate of the pdf of $X(t_2)$ given estimates of the pdfs of $X(t_1)$. This step is essentially a 2D translation and smoothing of the pdf of $X(t_1)$, as dictated by the velocity measurements and the statistics of error in these measurements, to obtain an estimate of the pdf of $X(t_2)$. The pdf of $X(t_1)$ is a weighted-cone (from iteration 2). Therefore, the message sent from g_2 to $X(t_2)$ is also a weighted-cone. A similar transformation is applied by g_2 to compute an estimate of the pdf of $X(t_1)$.

At the end of the third iteration, each of the state-variables $X(t_1)$ and $X(t_2)$ has two independent estimates of its pdf on its incoming links, which can be visualized as two different weighted-cones. As a final step, each state-variable intersects these pdfs to obtain a refined estimate of its pdf in 3D. This operation is visualized in Figure 8. The crucial point is that at the end of this step the possible values of the clock offset and position are constrained only to the regions where the cones intersect, consequently reducing the uncertainty in these estimates.

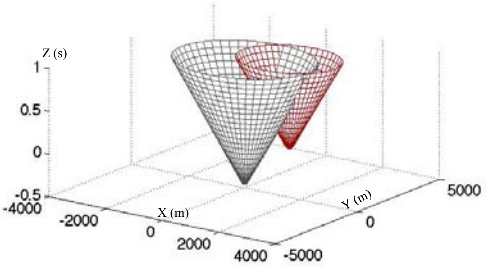


Figure 8: Reduction in the uncertainty of position and clock offset by intersecting two independently computed cones in 3D

6. SIMULATION RESULTS

In this section we compare the performance of our proposed scheme, ToA-TS with TDoA in simulations when the speed of the submersible and the interval between beacon transmissions are varied. We simulated a scenario consisting of four beacons and a submersible that moves in a 2 km x 2 km area. The trajectory of the submersible was generated using spline-interpolated waypoints. Transmissions from beacons were lagged by δt. We periodically tracked the position of the submersible using acoustic messages that were received over time epochs of duration $4\delta t$. For each time window, TDoA computes a single position estimate that corresponds to the middle of the window i.e. $2\delta t$. Our proposed scheme was used to obtain ML estimates of the position and the clock offset at the same time instances. We generated a sufficiently long trajectory for the submersible to capture the statistical variation in the RMS error of its position estimate due to its motion relative to the beacons. A random initial clock offset was added to the local time of the submersible in each simulation. The submersible's clock drift was chosen as 0.02 ppm, as reported in underwater systems that have integrated accurate clocks [4]. The simulation parameters are listed in Table 1.

Table 1: Simulation parameters

Area of deployment	2km x 2km
Signaling Interval, δt	0, 10, 20, 60 (s)
Speed, v	0.2, 0.8, 1.6 (m/s)
Std. of Velocity Error	0.2 m/s
Std. of Ranging Error	3 m
Clock Drift	0.02 ppm
Simulation Time	42.5 hrs

In a first set of simulations we compared the performance of the two schemes when signals were transmitted concurrently by beacons i.e. $\delta t = 0$. For this case, the histogram of the RMS error in position estimates for ToA-TS and TDoA are shown in Figure 9(a) and 9(b) respectively. As the figures show, the performance of the two schemes is nearly identical for this scenario. This is expected since the submersible's position does not change between signal arrivals. As such this is an ideal case for TDoA.

To evaluate the effect of motion on the tracking performance of both schemes we varied both the interval between beacon transmissions and the submersible's speed. Three sets of results are presented in Figures 10 to 12, each corresponding to a signaling interval of 10 s, 20 s and 60 s respectively. In each set, we compared the two schemes when the submersible's speed was varied as 0.2 m/s, 0.8 m/s and 1.6 m/s. The figures show the histogram in the RMS error in position estimates. The mean and standard deviation for each plot is marked at the top of each figure.

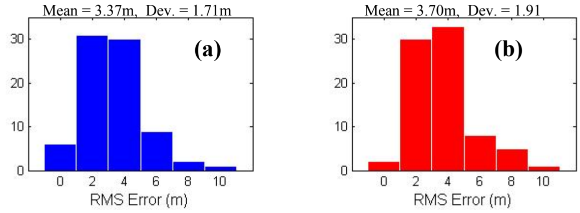


Figure 9: Histogram of the RMS Error in position estimates for (a) ToA-TS and (b) TDoA for concurrent beacon signals

To evaluate the effect of motion on the tracking performance of both schemes we varied both the interval between beacon transmissions and the submersible's speed. Three sets of results are presented in Figures 10 to 12, each corresponding to a signaling interval of 10 s, 20 s and 60 s respectively. In each set, we compared the two schemes when the submersible's speed was varied as 0.2 m/s, 0.8 m/s and 1.6 m/s. The figures show the histogram in the RMS error in position estimates. The mean and standard deviation for each plot is marked at the top of each figure.

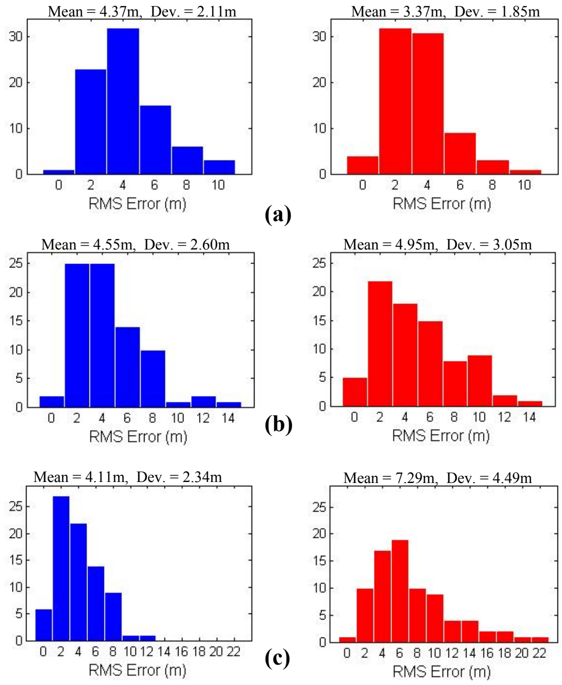


Figure 10: RMS error in position estimates for ToA-TS (left) and TDoA (right), signaling interval 10 s and submersible's speed equal to (a) 0.2 m/s (b) 0.8 m/s (c) 1.6 m/s

Figure 10(a)-(c) shows the performance of ToA-TS and TDoA for a fixed signaling interval of 10s and different submersible speeds. As shown in Figures 10(a)-(c), the performance of TDoA degrades as the speed increases, while that of our proposed method remains relatively the same. The mean and std. deviation of the RMS error for TDoA increases by a factor of two when the submersible's speed is increased from 0.2 m/s to 1.6 m/s. The performance of ToA-TS only shows a small statistical variation because it does not depend on the submersible's speed, rather on the error in velocity measurements, which was the same in all three cases. Our proposed scheme outperforms TDoA for speeds greater than 0.8 m/s, as observed in Figures 10(b) and 10(c). But, it does slightly worse than TDoA when the speed is 0.2 m/s, as shown in Figure 10(a). We attribute this to the fact that for this scenario the std. deviation of the error in velocity measurements is the same as the submersible's speed i.e. 0.2 m/s. These results indicate that using IMU measurements to account for the submersible's motion not useful if the submersible's IMU is not sufficiently accurate i.e. the errors in velocity estimates are comparable to or exceed the expected speed. However, TDoA does not always outperform our scheme when the error in velocity measurements is comparable to the speed. For example, the performance of TDoA is worse than ToA-TS, for the same speed and velocity error of 0.2 m/s, when the signaling interval is increased to a minute as shown in Figure 12(a).

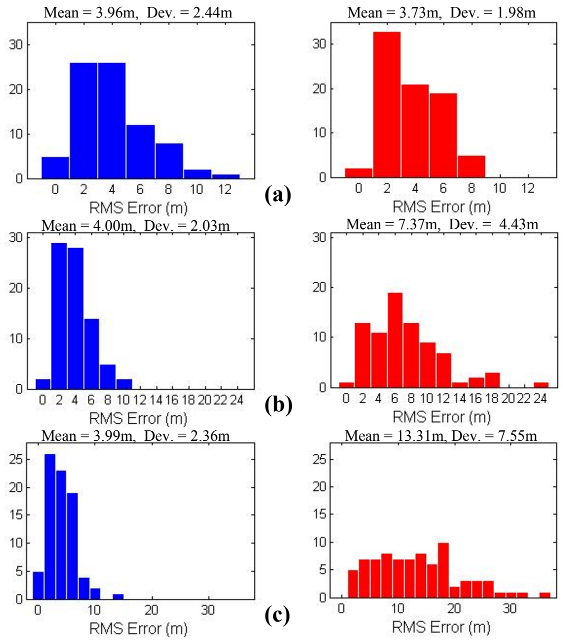


Figure 11: RMS error in position estimates for ToA-TS (left) and TDoA (right), signaling interval 20 s and submersible's speed equal to (a) 0.2 m/s (b) 0.8 m/s (c) 1.6 m/s

Next we compared the performance of the two schemes when the signaling interval was increased to 20 s and 60 s. The relative gain of using our method over TDoA becomes significant as the signaling interval is increased as shown in Figures 11 and 12. For a speed of 0.8 m/s, the mean RMS error of TDoA is 1.8 times that of ToA-TS for a signaling period of 20 s (Figure 11(b)) and 5 times that of ToA-TS for a signaling period of 60 s (Figure 12(b)). This corresponds to the case where the std. deviation of the velocity error is 25% of the submersible's speed. When the speed is increased to 1.6 m/s, the mean RMS error of TDoA is 3 and 8 times that of our method for signaling periods of 20 s and 60 s, respectively as shown in Figures 11(c) and 12(c). These results show that the localization performance can be significantly improved using ToA-TS.

The RMS error in the estimate of the clock offset using ToA-TS for signaling intervals of 10 s, 20 s and 60 s is shown in Figure 13 (a)-(c) respectively. The accuracy of the estimate in the clock offset is within 8 ms accuracy in all cases. The error increases as the signaling interval is increased. However, this is expected because of the error in velocity estimates that translate into an error in the estimated displacement. For a fixed velocity error, this error increases linearly with time. Since position and clock offset are jointly estimated, the error in velocity estimates also affects the accuracy of the clock offset estimates. The synchronization performance did not show variation with the submersible's speed for a fixed signaling interval and velocity error as expected.

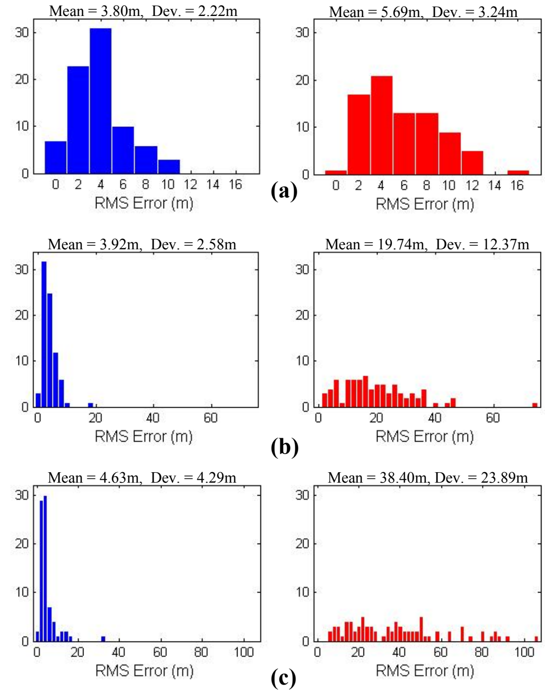
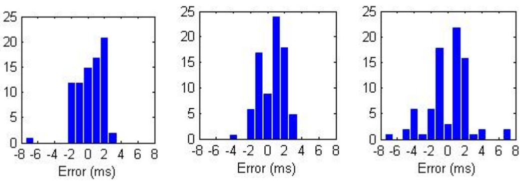


Figure 12: RMS error in position estimates for ToA-TS (left) and TDoA (right), signaling interval 60 s and submersible's speed equal to (a) 0.2 m/s (b) 0.8 m/s (c) 1.6 m/s



(a) (b) (c)

Figure 13: Histogram of error in estimated clock offset using ToA-TS when speed is 0.8 m/s and signaling interval is (a) 10 s (b) 20 s, (c) 60 s

Overall, our simulation results show that significant gains can be achieved by taking into account the submersible's motion between beacon transmissions compared to assuming that is stationary as in TDoA.

7. RELATED WORK

TDoA is one of the most widely used techniques to localize a submersible from one-way acoustic transmissions. Under ideal conditions, where transmissions from beacons occur concurrently, this technique serves as a way to both localize and time-synchronize the submersible from one-way acoustic transmissions. The TDoA approach has been extended to localizing submersible's in 3D where the depth of the vehicle is also unknown [10]. It has also been extended for the case where the beacons are not time-synchronized with each other [11]. However, these approaches do not consider the effect of vehicle motion for non-concurrent beacon transmissions.

Tracking and time-synchronization for mobile networks have been typically treated as separate problems [12][13]. A number of tracking techniques estimate a vehicle's position from non-concurrent distance estimates [8][14]. However, distance estimation requires round trip message exchange or prior time synchronization with the beacons. On the other hand, time-sync protocols that take vehicle motion into account, such as MU-Sync [15], D-Sync [16] and Mobi-Sync [17] require bi-directional acoustic signaling with beacons. Therefore, the signaling overhead associated with performing time-sync and tracking independently is substantial. In order to reduce the overall acoustic communication overhead compared to previous time synchronization protocols, JSL proposes to address the two problems jointly using a shared acoustic signaling scheme [18]. A key difference between JSL and our proposed method is that the former still uses a number of bi-directional message exchanges between the submersible and beacons while our proposed ToA-TS method is designed to work with only one-way beacon transmissions. The key motivation in the design of JSL is accounting for a depth dependent sound speed profile to correct estimates of propagation delay that are initially obtained from a straight line propagation model.

8. CONCLUSION

In this paper we have proposed ToA-TS, an approach that synchronizes a submersible while tracking it using only a few one-way acoustic transmissions. This method extends GPS-like time synchronization and localization for scenarios where beacon transmissions are no longer concurrent. We show that by using two simple hardware enhancements, namely an accurate clock-crystal and an on-board IMU, submersibles that can only receive acoustic transmissions can be accurately tracked and synchronized. Further, if submersibles have acoustic communication capability, our proposed approach can be applied for virtually no communication overhead on the part of submersibles.

9. ACKNOWLEDGMENTS

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10. REFERENCES

[1] K. Vickery, "Acoustic positioning systems. A practical overview of current systems", in *Proceedings of the Workshop on Autonomous Underwater Vehicles*(AUV'98), Cambridge, MA, USA. pp. 5-17, August 1998.

[2] D. Mirza, P. Roberts, J.Yi, C. Schurgers, R. Kastner, J. Jaffe, "Energy Efficient Signaling Strategies for Tracking Mobile Underwater Vehicles", Underwater Technology, 2013.

[3] D. Mirza, C. Schurgers, "Energy-efficient ranging for post-facto self-localization in mobile underwater networks", IEEE Journal on Selected Areas in Communications, Vol. 26, No. 9, 2008.

[4] R. M. Eustice, L. L. Whitcomb, H. Singh, M. Grund, "Recent advances in synchronous-clock one-way-travel-time acoustic navigation", *Oceans*'06, 2006.

[5] T. Schmid, J. Friedman, Z. Charbiwala, Y.H. Cho, M. Srivastava, "XCXO: an ultra-low cost ultra-high accuracy clock system for wireless sensor networks in harsh remote outdoor environments", ISSCC/DAC'08, 2008.

[6] F.R. Kschischang, B.J. Frey, H.A. Loeliger, "Factor graphs and the sum-product algorithm", IEEE Trans. on Information Theory, Vol. 47, No. 2, 2001.

[7] H.A. Loeliger, "An introduction to factor graphs", Signal Processing Magazine, Vol. 21, No. 1, 2004.

[8] D. Mirza, C. Schurgers, R. Kastner, "Real-time collaborative tracking for underwater networked systems", *WUWNet*'12, 2012.

[9] A. Caiti, A. Garulli, F. Livide, D. Prattichizzo, "Localization of autonomous underwater vehicles by floating acoustic buoys: a set-membership approach", IEEE Journal of Oceanic Engineering, Vol. 30, No. 1, 2005.

[10] C. Tian, W. Liu, J. Jin, Y. Wang, "Localization and Synchronization for 3D Underwater Acoustic Sensor Networks", Springer-Verlag Berlin Heidelbery, 2007.

[11] X. Cheng, H. Shu, Q. Liang, D.C. Du, "Silent Positioning in Underwater Acoustic Sensor Networks", IEEE Trans. Vehicular Technology, Vol. 57, No. 3, pp. 1756-1766, 2008.

[12] H.P. Tan, R Diamant, W.K.G. Seah,"A survey of techniques and challenges in underwater localization", Journal of Ocean Engineering, Vol. 38, pp. 1663-1676, 2011.

[13] F. Sivrikaya, B. Yener, "Time synchronization in sensor networks: A survey", IEEE Network, Vol. 18, 2004.

[14] D. Fox, W. Burgard, H Kruppa, S. Thrun, "A Probabilistic Approach to Collaborative Multi-Robot Localization", Autonomous Robots, Vol. 8, No.3, pp. 325-344, 2000.

[15] N. Chirdchoo, W.-S. Soh, K. C. Chua, "MU-Sync: A time synchronization protocol for underwater mobile networks", *WUWNet'08,* 2008.

[16] F. Lu, D. Mirza, C. Schurgers, "D-Sync: doppler-based time synchronization for mobile underwater sensor networks", *WUWNet*'10, 2010.

[17] J. Liu, Z. Zhou, Z. Peng, and J.-H. Cui, "Mobi-sync: Efficient time synchronization for mobile underwater sensor networks", in *Proceedings of IEEE Globecom,* December 2010.

[18] J. Liu, Z. Wang, M. Zuba, Z. Peng, J.H. Cui, S. Zhou, "JSL: joint time synchronization and localization design with stratification compensation in mobile underwater sensor networks", *SECON*'12, 2012.