Multiple Choice Questions for Review

Some of the following questions assume that you have done the exercises.

1. Indicate which, if any, of the following five graphs $G = (V, E, \phi)$, $|V| = 5$, is not isomorphic to any of the other four.

(a) $\phi = \begin{pmatrix} A & B & C & D & E & F \\ \{1,3\} & \{2,4\} & \{1,2\} & \{2,3\} & \{3,5\} & \{4,5\} \end{pmatrix}$

(b) $\phi = \begin{pmatrix} f & b & c & d & e & a \\ \{1,2\} & \{1,2\} & \{2,3\} & \{3,4\} & \{3,4\} & \{4,5\} \end{pmatrix}$

(c) $\phi = \begin{pmatrix} b & f & e & d & c & a \\ \{4,5\} & \{1,3\} & \{1,3\} & \{2,3\} & \{2,4\} & \{4,5\} \end{pmatrix}$

(d) $\phi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \{1,2\} & \{2,3\} & \{2,3\} & \{3,4\} & \{4,5\} & \{4,5\} \end{pmatrix}$

(e) $\phi = \begin{pmatrix} b & a & e & d & c & f \\ \{4,5\} & \{1,3\} & \{1,3\} & \{2,3\} & \{2,5\} & \{4,5\} \end{pmatrix}$

2. Indicate which, if any, of the following five graphs $G = (V, E, \phi)$, $|V| = 5$, is not connected.

(a) $\phi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \{1,2\} & \{1,2\} & \{2,3\} & \{3,4\} & \{1,5\} & \{1,5\} \end{pmatrix}$

(b) $\phi = \begin{pmatrix} b & a & e & d & c & f \\ \{4,5\} & \{1,3\} & \{1,3\} & \{2,3\} & \{2,5\} & \{4,5\} \end{pmatrix}$

(c) $\phi = \begin{pmatrix} b & f & e & d & c & a \\ \{4,5\} & \{1,3\} & \{1,3\} & \{2,3\} & \{2,4\} & \{4,5\} \end{pmatrix}$

(d) $\phi = \begin{pmatrix} a & b & c & d & e & f \\ \{1,2\} & \{2,3\} & \{1,2\} & \{2,3\} & \{3,4\} & \{1,5\} \end{pmatrix}$

(e) $\phi = \begin{pmatrix} a & b & c & d & e & f \\ \{1,2\} & \{2,3\} & \{1,2\} & \{1,3\} & \{2,3\} & \{4,5\} \end{pmatrix}$

3. Indicate which, if any, of the following five graphs $G = (V, E, \phi)$, $|V| = 5$, have an Eulerian circuit.

(a) $\phi = \begin{pmatrix} F & B & C & D & E & A \\ \{1,2\} & \{1,2\} & \{2,3\} & \{3,4\} & \{4,5\} & \{4,5\} \end{pmatrix}$

(b) $\phi = \begin{pmatrix} b & f & e & d & c & a \\ \{4,5\} & \{1,3\} & \{1,3\} & \{2,3\} & \{2,4\} & \{4,5\} \end{pmatrix}$

(c) $\phi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \{1,2\} & \{1,2\} & \{2,3\} & \{3,4\} & \{4,5\} & \{4,5\} \end{pmatrix}$

(d) $\phi = \begin{pmatrix} b & a & e & d & c & f \\ \{4,5\} & \{1,3\} & \{1,3\} & \{2,3\} & \{2,5\} & \{4,5\} \end{pmatrix}$

(e) $\phi = \begin{pmatrix} a & b & c & d & e & f \\ \{1,3\} & \{3,4\} & \{1,2\} & \{2,3\} & \{3,5\} & \{4,5\} \end{pmatrix}$

4. A graph with $V = \{1, 2, 3, 4\}$ is described by $\phi = \begin{pmatrix} a & b & c & d & e & f \\ \{1,2\} & \{1,2\} & \{1,4\} & \{2,3\} & \{3,4\} & \{3,4\} \end{pmatrix}$. How many Hamiltonian cycles does it have?

(a) 1 (b) 2 (c) 4 (d) 16 (e) 32

5. A graph with $V = \{1,2,3,4\}$ is described by $\phi = \begin{pmatrix} a & b & c & d & e & f \\ \{1,2\} & \{1,2\} & \{1,4\} & \{2,3\} & \{3,4\} & \{3,4\} \end{pmatrix}$. It has weights on its edges given by $\lambda = \begin{pmatrix} a & b & c & d & e & f \\ 3 & 2 & 1 & 2 & 4 & 2 \end{pmatrix}$. How many minimum spanning trees does it have?

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

6. Define an RP-tree by the parent-child adjacency lists as follows:

(i) Root B: J, H, K; (ii) H: P, Q, R; (iii) Q: S, T; (iv) K: L, M, N.

The postorder vertex sequence of this tree is

(a) J, P, S, T, Q, R, H, L, M, N, K, B.

(b) P, S, T, J, Q, R, H, L, M, N, K, B.

(c) P, S, T, Q, R, H, L, M, N, K, J, B.

(d) P, S, T, Q, R, J, H, L, M, N, K, B.

(e) S, T, Q, J, P, R, H, L, M, N, K, B.

7. Define an RP-tree by the parent-child adjacency lists as follows:

(i) Root B: J, H, K; (ii) J: P, Q, R; (iii) Q: S, T; (iv) K: L, M, N.

The preorder vertex sequence of this tree is

(a) B, J, H, K, P, Q, R, L, M, N, S, T.

(b) B, J, P, Q, S, T, R, H, K, L, M, N.

(c) B, J, P, Q, S, T, R, H, L, M, N, K.

(d) B, J, Q, P, S, T, R, H, L, M, N, K.

(e) B, J, Q, S, T, P, R, H, K, L, M, N.

8. For which of the following does there exist a graph $G = (V, E, \phi)$ satisfying the specified conditions?

(a) A tree with 9 vertices and the sum of the degrees of all the vertices is 18.

(b) A graph with 5 components 12 vertices and 7 edges.

(c) A graph with 5 components 30 vertices and 24 edges.

(d) A graph with 9 vertices, 9 edges, and no cycles.

(e) A connected graph with 12 edges 5 vertices and fewer than 8 cycles.

9. For which of the following does there exist a simple graph $G = (V, E)$ satisfying the specified conditions?

(a) It has 3 components 20 vertices and 16 edges.

(b) It has 6 vertices, 11 edges, and more than one component.

(c) It is connected and has 10 edges 5 vertices and fewer than 6 cycles.

(d) It has 7 vertices, 10 edges, and more than two components.

(e) It has 8 vertices, 8 edges, and no cycles.

10. For which of the following does there exist a tree satisfying the specified constraints?

(a) A binary tree with 65 leaves and height 6.

(b) A binary tree with 33 leaves and height 5.

(c) A full binary tree with height 5 and 64 total vertices.

(d) A full binary tree with 23 leaves and height 23.

(e) A rooted tree of height 3, every vertex has at most 3 children. There are 40 total vertices.

11. For which of the following does there exist a tree satisfying the specified constraints?

(a) A full binary tree with 31 leaves, each leaf of height 5.

(b) A rooted tree of height 3 where every vertex has at most 3 children and there are 41 total vertices.

(c) A full binary tree with 11 vertices and height 6.

(d) A binary tree with 2 leaves and height 100.

(e) A full binary tree with 20 vertices.

12. The number of simple digraphs with $|V| = 3$ is

(a) 2^9 (b) 2^8 (c) 2^7 (d) 2^6 (e) 2^5

13. The number of simple digraphs with $|V| = 3$ and exactly 3 edges is

(a) 92 (b) 88 (c) 80 (d) 84 (e) 76

14. The number of oriented simple graphs with $|V| = 3$ is

(a) 27 (b) 24 (c) 21 (d) 18 (e) 15

15. The number of oriented simple graphs with $|V| = 4$ and 2 edges is

(a) 40 (b) 50 (c) 60 (d) 70 (e) 80

16. In each case the depth-first sequence of an ordered rooted spanning tree for a graph G is given. Also given are the non-tree edges of G. Which of these spanning trees is a depth-first spanning tree?

(a) 123242151 and $\{3, 4\}$, $\{1, 4\}$

(b) 123242151 and $\{4, 5\}$, $\{1, 3\}$

(c) 123245421 and $\{2, 5\}$, $\{1, 4\}$

(d) 123245421 and $\{3, 4\}$, $\{1, 4\}$

(e) 123245421 and $\{3, 5\}$, $\{1, 4\}$

17. $\sum_{i=1}^{n} i^{-1/2}$ is

(a) $\Theta((\ln(n))^{1/2})$ (b) $\Theta(\ln(n))$ (c) $\Theta(n^{1/2})$ (d) $\Theta(n^{3/2})$ (e) $\Theta(n^2)$

18. Compute the total number of bicomponents in all of the following three simple graphs, $G = (V, E)$ with $|V| = 5$. For each graph the edge sets are as follows:

$$E = \big\{\{1,2\},\ \{2,3\},\ \{3,4\},\ \{4,5\},\ \{1,3\},\ \{1,5\},\ \{3,5\}\big\}$$

$$E = \big\{\{1,2\},\ \{2,3\},\ \{3,4\},\ \{4,5\},\ \{1,3\}\big\}$$

$$E = \big\{\{1,2\},\ \{2,3\},\ \{4,5\},\ \{1,3\}\big\}$$

(a) 4 (b) 5 (c) 6 (d) 7 (e) 8

19. Let $b > 1$. Then $\log_b((n^2)!)$ is

(a) $\Theta(\log_b(n!))$

(b) $\Theta(\log_b(2\,n!))$

(c) $\Theta(n \log_b(n))$

(d) $\Theta(n^2 \log_b(n))$

(e) $\Theta(n \log_b(n^2))$

20. What is the total number of additions and multiplications in the following code?

```
s := 0
for i := 1 to n
    s:= s + i
    for j:= 1 to i
        s := s + j*i
    next j
next i
s := s+10
```

(a) n (b) n^2 (c) $n^2 + 2n$ (d) $n(n+1)$ (e) $(n+1)^2$

Answers: **1** (a), **2** (e), **3** (e), **4** (c), **5** (b), **6** (a), **7** (b), **8** (b), **9** (d), **10** (e), **11** (d), **12** (a), **13** (d), **14** (a), **15** (c), **16** (c), **17** (c), **18** (c), **19** (d), **20** (e).

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