

Multiple Choice Questions for Review

Some of the following questions assume that you have done the exercises.

1. Indicate which, if any, of the following five graphs $G = (V, E, \phi)$, $|V| = 5$, is not isomorphic to any of the other four.

$$(a) \phi = \begin{pmatrix} A & B & C & D & E & F \\ \{1,3\} & \{2,4\} & \{1,2\} & \{2,3\} & \{3,5\} & \{4,5\} \end{pmatrix}$$

$$(b) \phi = \begin{pmatrix} f & b & c & d & e & a \\ \{1,2\} & \{1,2\} & \{2,3\} & \{3,4\} & \{3,4\} & \{4,5\} \end{pmatrix}$$

$$(c) \phi = \begin{pmatrix} b & f & e & d & c & a \\ \{4,5\} & \{1,3\} & \{1,3\} & \{2,3\} & \{2,4\} & \{4,5\} \end{pmatrix}$$

$$(d) \phi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \{1,2\} & \{2,3\} & \{2,3\} & \{3,4\} & \{4,5\} & \{4,5\} \end{pmatrix}$$

$$(e) \phi = \begin{pmatrix} b & a & e & d & c & f \\ \{4,5\} & \{1,3\} & \{1,3\} & \{2,3\} & \{2,5\} & \{4,5\} \end{pmatrix}$$

2. Indicate which, if any, of the following five graphs $G = (V, E, \phi)$, $|V| = 5$, is not connected.

$$(a) \phi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \{1,2\} & \{1,2\} & \{2,3\} & \{3,4\} & \{1,5\} & \{1,5\} \end{pmatrix}$$

$$(b) \phi = \begin{pmatrix} b & a & e & d & c & f \\ \{4,5\} & \{1,3\} & \{1,3\} & \{2,3\} & \{2,5\} & \{4,5\} \end{pmatrix}$$

$$(c) \phi = \begin{pmatrix} b & f & e & d & c & a \\ \{4,5\} & \{1,3\} & \{1,3\} & \{2,3\} & \{2,4\} & \{4,5\} \end{pmatrix}$$

$$(d) \phi = \begin{pmatrix} a & b & c & d & e & f \\ \{1,2\} & \{2,3\} & \{1,2\} & \{2,3\} & \{3,4\} & \{1,5\} \end{pmatrix}$$

$$(e) \phi = \begin{pmatrix} a & b & c & d & e & f \\ \{1,2\} & \{2,3\} & \{1,2\} & \{1,3\} & \{2,3\} & \{4,5\} \end{pmatrix}$$

3. Indicate which, if any, of the following five graphs $G = (V, E, \phi)$, $|V| = 5$, have an Eulerian circuit.

$$(a) \phi = \begin{pmatrix} F & B & C & D & E & A \\ \{1,2\} & \{1,2\} & \{2,3\} & \{3,4\} & \{4,5\} & \{4,5\} \end{pmatrix}$$

$$(b) \phi = \begin{pmatrix} b & f & e & d & c & a \\ \{4,5\} & \{1,3\} & \{1,3\} & \{2,3\} & \{2,4\} & \{4,5\} \end{pmatrix}$$

$$(c) \phi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \{1,2\} & \{1,2\} & \{2,3\} & \{3,4\} & \{4,5\} & \{4,5\} \end{pmatrix}$$

$$(d) \phi = \begin{pmatrix} b & a & e & d & c & f \\ \{4,5\} & \{1,3\} & \{1,3\} & \{2,3\} & \{2,5\} & \{4,5\} \end{pmatrix}$$

$$(e) \phi = \begin{pmatrix} a & b & c & d & e & f \\ \{1,3\} & \{3,4\} & \{1,2\} & \{2,3\} & \{3,5\} & \{4,5\} \end{pmatrix}$$

4. A graph with $V = \{1, 2, 3, 4\}$ is described by $\phi = \begin{pmatrix} a & b & c & d & e & f \\ \{1,2\} & \{1,2\} & \{1,4\} & \{2,3\} & \{3,4\} & \{3,4\} \end{pmatrix}$. How many Hamiltonian cycles does it have?

- (a) 1 (b) 2 (c) 4 (d) 16 (e) 32

Review Questions

5. A graph with $V = \{1, 2, 3, 4\}$ is described by $\phi = \left(\begin{array}{cccccc} a & b & c & d & e & f \\ \{1,2\} & \{1,2\} & \{1,4\} & \{2,3\} & \{3,4\} & \{3,4\} \end{array} \right)$. It has weights on its edges given by $\lambda = \left(\begin{array}{cccccc} a & b & c & d & e & f \\ 3 & 2 & 1 & 2 & 4 & 2 \end{array} \right)$. How many minimum spanning trees does it have?

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

6. Define an RP-tree by the parent-child adjacency lists as follows:

(i) Root B: J, H, K; (ii) H: P, Q, R; (iii) Q: S, T; (iv) K: L, M, N.

The postorder vertex sequence of this tree is

- (a) J, P, S, T, Q, R, H, L, M, N, K, B.
- (b) P, S, T, J, Q, R, H, L, M, N, K, B.
- (c) P, S, T, Q, R, H, L, M, N, K, J, B.
- (d) P, S, T, Q, R, J, H, L, M, N, K, B.
- (e) S, T, Q, J, P, R, H, L, M, N, K, B.

7. Define an RP-tree by the parent-child adjacency lists as follows:

(i) Root B: J, H, K; (ii) J: P, Q, R; (iii) Q: S, T; (iv) K: L, M, N.

The preorder vertex sequence of this tree is

- (a) B, J, H, K, P, Q, R, L, M, N, S, T.
- (b) B, J, P, Q, S, T, R, H, K, L, M, N.
- (c) B, J, P, Q, S, T, R, H, L, M, N, K.
- (d) B, J, Q, P, S, T, R, H, L, M, N, K.
- (e) B, J, Q, S, T, P, R, H, K, L, M, N.

8. For which of the following does there exist a graph $G = (V, E, \phi)$ satisfying the specified conditions?

- (a) A tree with 9 vertices and the sum of the degrees of all the vertices is 18.
- (b) A graph with 5 components 12 vertices and 7 edges.
- (c) A graph with 5 components 30 vertices and 24 edges.
- (d) A graph with 9 vertices, 9 edges, and no cycles.
- (e) A connected graph with 12 edges 5 vertices and fewer than 8 cycles.

9. For which of the following does there exist a simple graph $G = (V, E)$ satisfying the specified conditions?

- (a) It has 3 components 20 vertices and 16 edges.
- (b) It has 6 vertices, 11 edges, and more than one component.

Basic Concepts in Graph Theory

- (c) It is connected and has 10 edges 5 vertices and fewer than 6 cycles.
 - (d) It has 7 vertices, 10 edges, and more than two components.
 - (e) It has 8 vertices, 8 edges, and no cycles.
10. For which of the following does there exist a tree satisfying the specified constraints?
- (a) A binary tree with 65 leaves and height 6.
 - (b) A binary tree with 33 leaves and height 5.
 - (c) A full binary tree with height 5 and 64 total vertices.
 - (d) A full binary tree with 23 leaves and height 23.
 - (e) A rooted tree of height 3, every vertex has at most 3 children. There are 40 total vertices.
11. For which of the following does there exist a tree satisfying the specified constraints?
- (a) A full binary tree with 31 leaves, each leaf of height 5.
 - (b) A rooted tree of height 3 where every vertex has at most 3 children and there are 41 total vertices.
 - (c) A full binary tree with 11 vertices and height 6.
 - (d) A binary tree with 2 leaves and height 100.
 - (e) A full binary tree with 20 vertices.
12. The number of simple digraphs with $|V| = 3$ is
- (a) 2^9 (b) 2^8 (c) 2^7 (d) 2^6 (e) 2^5
13. The number of simple digraphs with $|V| = 3$ and exactly 3 edges is
- (a) 92 (b) 88 (c) 80 (d) 84 (e) 76
14. The number of oriented simple graphs with $|V| = 3$ is
- (a) 27 (b) 24 (c) 21 (d) 18 (e) 15
15. The number of oriented simple graphs with $|V| = 4$ and 2 edges is
- (a) 40 (b) 50 (c) 60 (d) 70 (e) 80
16. In each case the depth-first sequence of an ordered rooted spanning tree for a graph G is given. Also given are the non-tree edges of G . Which of these spanning trees is a depth-first spanning tree?
- (a) 123242151 and $\{3, 4\}, \{1, 4\}$
 - (b) 123242151 and $\{4, 5\}, \{1, 3\}$
 - (c) 123245421 and $\{2, 5\}, \{1, 4\}$
 - (d) 123245421 and $\{3, 4\}, \{1, 4\}$
 - (e) 123245421 and $\{3, 5\}, \{1, 4\}$

Review Questions

17. $\sum_{i=1}^n i^{-1/2}$ is
 (a) $\Theta((\ln(n))^{1/2})$ (b) $\Theta(\ln(n))$ (c) $\Theta(n^{1/2})$ (d) $\Theta(n^{3/2})$ (e) $\Theta(n^2)$
18. Compute the total number of bicomponents in all of the following three simple graphs, $G = (V, E)$ with $|V| = 5$. For each graph the edge sets are as follows:
 $E = \{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{4, 5\}, \{1, 3\}, \{1, 5\}, \{3, 5\}\}$
 $E = \{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{4, 5\}, \{1, 3\}\}$
 $E = \{\{1, 2\}, \{2, 3\}, \{4, 5\}, \{1, 3\}\}$
 (a) 4 (b) 5 (c) 6 (d) 7 (e) 8
19. Let $b > 1$. Then $\log_b((n^2)!)$ is
 (a) $\Theta(\log_b(n!))$
 (b) $\Theta(\log_b(2n!))$
 (c) $\Theta(n \log_b(n))$
 (d) $\Theta(n^2 \log_b(n))$
 (e) $\Theta(n \log_b(n^2))$
20. What is the total number of additions and multiplications in the following code?

```

s := 0
for i := 1 to n
    s := s + i
    for j := 1 to i
        s := s + j*i
    next j
next i
s := s+10
    
```

- (a) n (b) n^2 (c) $n^2 + 2n$ (d) $n(n + 1)$ (e) $(n + 1)^2$

Answers: 1 (a), 2 (e), 3 (e), 4 (c), 5 (b), 6 (a), 7 (b), 8 (b), 9 (d), 10 (e), 11 (d), 12 (a), 13 (d), 14 (a), 15 (c), 16 (c), 17 (c), 18 (c), 19 (d), 20 (e).

Notation Index

$s \sim t$ (equivalence relation) GT-5

BFE(T) (breadth first vertex
sequence) GT-29

BFV(T) (breadth first vertex
sequence) GT-29

DFV(T) (depth first vertex
sequence) GT-29

$x|y$ (x divides y) GT-24

DFE(T) (depth first edge
sequence) GT-29

(V, E) (simple graph) GT-2

(V, E, ϕ) (graph) GT-3

$O()$ (Big oh notation) GT-38

$o()$ (little oh notation) GT-40

$\Theta()$ (rate of growth) GT-38

Subject Index

- Adjacent vertices GT-3
- Algorithm
 - divide and conquer GT-45
 - Kruskal's (minimum weight spanning tree) GT-33
 - lineal (= depth-first) spanning tree GT-33
 - partial GT-45
 - polynomial time (tractable) GT-43
 - Prim's (minimum weight spanning tree) GT-32
 - which is faster? GT-43
- Antisymmetric binary relation GT-24
- Asymptotic GT-40
- Average running time GT-42

- Bicomponents GT-22
- Biconnected components GT-22
- Binary relation GT-5
 - antisymmetric GT-24
 - covering GT-24
 - equivalence relation GT-5
 - order relation GT-24
 - reflexive GT-5
 - symmetric GT-5
 - transitive GT-5
- Binary tree GT-36
 - full GT-36
- Bipartite graph GT-23
 - cycle lengths of GT-34
- Breadth first vertex (edge) sequence GT-29

- Child vertex GT-27
- Chromatic number GT-42, GT-45
- Circuit in a graph GT-18
 - Eulerian GT-21
- Clique GT-44
- Clique problem GT-44
- Coloring a graph GT-42, GT-45
- Coloring problem GT-44
- Comparing algorithms GT-43
- Complete simple graph GT-16
- Component connected GT-19
- Connected components GT-19
- Covering relation GT-24
- Cycle in a graph GT-18
 - Hamiltonian GT-21

- Decision tree
 - see also* Rooted tree
 - ordered tree is equivalent GT-27
 - RP-tree is equivalent GT-27
 - traversals GT-28
- Degree of a vertex GT-4
- Degree sequence of a graph GT-4
- Depth first vertex (edge) sequence GT-29
- Digraph GT-15
 - functional GT-30
- Directed graph GT-15
- Directed loop GT-15
- Divide and conquer GT-45

- Edge GT-2
 - directed GT-15
 - incident on vertex GT-3
 - loop GT-4, GT-11
 - parallel GT-11
- Edge sequence
 - breadth first GT-29
 - depth first GT-29
- Equivalence class GT-5
- Equivalence relation GT-5

Index

- Eulerian circuit or trail GT-21
- Full binary tree GT-36
- Graph GT-3
 - see also* specific topic
 - biconnected GT-22
 - bipartite GT-23
 - bipartite and cycle
 - lengths GT-34
 - complete simple GT-16
 - connected GT-19, GT-19
 - directed GT-15
 - incidence function GT-3
 - induced subgraph (by edges or vertices) GT-18
 - isomorphism GT-7
 - oriented simple GT-24
 - random GT-8
 - rooted GT-27
 - simple GT-2
 - subgraph of GT-17
- Growth
 - rate of, *see* Rate of growth
- Hamiltonian cycle GT-21
- Hasse diagram GT-24
- Height of a tree GT-36
- Incidence function of a graph GT-3
- Induced subgraph (by edges or vertices) GT-18
- Internal vertex GT-27
- Isolated vertex GT-11
- Isomorphic graphs GT-7
- Kruskal's algorithm for minimum weight spanning tree GT-33
- Leaf vertex GT-27
- Little oh notation GT-40
- Loop GT-4, GT-11
 - directed GT-15
- Machine independence GT-38
- Merge sorting GT-46
- NP-complete problem GT-44
- NP-easy problem GT-44
- NP-hard problem GT-44
- Order relation GT-24
- Oriented simple graph GT-24
- Parallel edges GT-11
- Parent vertex GT-27
- Path in a (directed) graph GT-16
- Polynomial multiplication GT-48
- Polynomial time algorithm (tractable) GT-43
- Prim's algorithm for minimum weight spanning tree GT-32
- Random graphs GT-8
- Rate of growth
 - Big oh notation GT-38
 - comparing GT-43
 - exponential GT-43
 - little oh notation GT-40
 - polynomial GT-40, GT-43
 - Theta notation GT-38
- Reflexive relation GT-5
- Relation
 - see perhaps* Binary relation
- Rooted graph GT-27

- Rooted tree
 - child GT-27
 - internal vertex GT-27
 - leaf GT-27
 - parent GT-27
 - siblings GT-27
- RP-tree (rooted plane tree)
 - see* Decision tree
- Simple graph GT-2
- Sorting (merge sort) GT-46
- Spanning tree GT-31
 - lineal (= depth first) GT-34
 - minimum weight GT-31
- Subgraph GT-17
 - cycle GT-18
 - induced by edges or vertices GT-18
- Symmetric relation GT-5
- Theorem
 - bipartite and cycle lengths GT-34
 - cycles and multiple paths GT-19
 - equivalence relations GT-5
 - minimum weight spanning tree GT-32
 - Prim's algorithm GT-32
 - properties of Θ and O GT-39
 - walk, trail and path GT-17
- Tractable algorithm GT-44
- Trail in a (directed) graph GT-16
- Transitive relation GT-5
- Traveling salesman problem GT-44
- Traversal
 - decision tree GT-28
- Tree
 - see also* specific topic
 - binary GT-36
 - decision, *see* Decision tree
 - height GT-36
 - ordered tree, *see* Decision tree
 - rooted, *see* Rooted tree
 - RP-tree (rooted plane tree), *see* Decision tree
 - spanning GT-31
 - spanning, lineal (= depth first) GT-34
 - spanning, minimum weight GT-31
- Vertex
 - adjacent pair GT-3
 - child GT-27
 - degree of GT-4
 - internal GT-27
 - isolated GT-11
 - leaf GT-27
 - parent GT-27
- Vertex sequence GT-16
 - breadth first GT-29
 - depth first GT-29
- Walk in a graph GT-16