

Multiple Choice Questions for Review

- In each case, two permutations on $\underline{6}$ are listed. In which case is the first permutation less than the second in direct insertion order?
 - 2, 3, 1, 4, 5, 6 1, 3, 2, 4, 5, 6
 - 2, 3, 1, 4, 5, 6 2, 1, 3, 4, 5, 6
 - 2, 3, 1, 4, 5, 6 4, 5, 6, 1, 3, 2
 - 6, 1, 2, 3, 4, 5 2, 1, 3, 4, 5, 6
 - 6, 2, 3, 1, 4, 5 2, 3, 1, 4, 5, 6
- What is the rank, in direct insertion order, of the permutation 5, 4, 6, 3, 2, 1?
 - 3 (b) 4 (c) 715 (d) 716 (e) 717
- What is the rank, in lex order, of the permutation 6, 1, 2, 3, 4, 5?
 - 20 (b) 30 (c) 480 (d) 600 (e) 619
- Consider the list of all sequences of length six of A's and B's that satisfy the following conditions:
 - There are no two adjacent A's.
 - There are never three B's adjacent.What is the next sequence after ABBABB in lex order?
 - ABABAB
 - ABBABA
 - BABABA
 - BABBAB
 - BBABBA
- Which of the following 4×4 domino covers represent two distinct hibachi grills?
 - hhhhhvh and hvhnhhh
 - hvhvvh and vvhhvvh
 - vhvvvh and hhvvhv
 - vvhhvvh and hhhhvvh
 - vvvvvv and hhhhhhh
- Given that $a_0 = 1$, $a_n = n + (-1)^n a_{n-1}$ for $n \geq 1$. What is the value of a_4 ?
 - 1 (b) 4 (c) 5 (d) 8 (e) 11
- Given that $a_k = a_{k-1}/(1 + a_{k-1})$ for $k \geq 1$, $a_0 = 1$. Which of the following gives an explicit formula for a_k ?
 - $1/3^k$, $k = 0, 1, 2, 3, \dots$

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- (b) $1/2^k$, $k = 0, 1, 2, 3, \dots$
- (c) $1/(3^{k+1} - 2)$, $k = 0, 1, 2, 3, \dots$
- (d) $1/(k + 1)$, $k = 0, 1, 2, 3, \dots$
- (e) $2/(k + 2)$, $k = 0, 1, 2, 3, \dots$
8. Consider the recurrence relation $a_k = -8a_{k-1} - 15a_{k-2}$ with initial conditions $a_0 = 0$ and $a_1 = 2$. Which of the following is an explicit solution to this recurrence relation?
- (a) $a_k = (-3)^k - (-5)^k$
- (b) $a_k = k(-3)^k - k(-5)^k$
- (c) $a_k = k(-3)^k - (-5)^k$
- (d) $a_k = (-5)^k - (-3)^k$
- (e) $a_k = k(-5)^k - k(-3)^k$
9. Consider the recurrence relation $a_k = 6a_{k-1} - 9a_{k-2}$ with initial conditions $a_0 = 0$ and $a_1 = 2$. Which of the following is an explicit solution to this recurrence relation, provided the constants A and B are chosen correctly?
- (a) $a_n = A3^n + B3^n$
- (b) $a_n = A3^n + B(-3)^n$
- (c) $a_n = A3^n + nB3^n$
- (d) $a_n = A(-3)^n + nB(-3)^n$
- (e) $a_n = nA3^n + nB3^n$
10. In the Towers of Hanoi puzzle $H(8, S, E, G)$, the configuration is

Pole S: 6, 5; Pole E: 1; Pole G: 8,7,4,3,2.

What move was just made to create this configuration?

- (a) washer 1 from S to E
- (b) washer 1 from G to E
- (c) washer 2 from S to G
- (d) washer 2 from E to G
- (e) washer 5 from G to S
11. In the Towers of Hanoi puzzle $H(8, S, E, G)$, the configuration is

Pole S: 6, 5; Pole E: empty; Pole G: 8, 7, 4, 3, 2, 1.

What are the next two moves?

- (a) washer 1 from G to E followed by washer 2 from G to S
- (b) washer 1 from G to S followed by washer 2 from G to E
- (c) washer 5 from S to E followed by washer 1 from G to E

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- (d) washer 5 from S to E followed by washer 1 from G to S
(e) washer 5 from S to E followed by washer 2 from G to S
- 12.** In the Towers of Hanoi puzzle $H(8, S, E, G)$, the configuration is
- Pole S: 6, 5, 2; Pole E: 1; Pole G: 8, 7, 4, 3.
- The next move is washer 2 from S to G. What is the RANK of this move in the list of all moves for $H(8, S, E, G)$?
- (a) 205 (b) 206 (c) 214 (d) 215 (e) 216
- 13.** In the subset Gray code for $n = 6$, what is the next element after 111000?
- (a) 000111
(b) 101000
(c) 111001
(d) 111100
(e) 101100
- 14.** In the subset Gray code for $n = 6$, what is the element just before 110000?
- (a) 010000
(b) 100000
(c) 110001
(d) 110100
(e) 111000
- 15.** In the subset Gray code for $n = 6$, what is the RANK of 110000?
- (a) 8 (b) 16 (c) 32 (d) 48 (e) 63
- 16.** In the subset Gray code for $n = 6$, what is the element of RANK 52?
- (a) 101011
(b) 101110
(c) 101101
(d) 110000
(e) 111000
- 17.** The probability of team A winning any game is $1/3$. Team A plays team B in a tournament. If either team wins two games in a row, that team is declared the winner. At most three games are played in the tournament and, if no team has won the tournament at the end of three games, the tournament is declared a draw. What is the expected number of games in the tournament?
- (a) 3 (b) $19/9$ (c) $22/9$ (d) $25/9$ (e) $61/27$
- 18.** The probability of team A winning any game is $1/2$. Team A plays team B in a tournament. If either team wins two games in a row, that team is declared the winner. At

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most four games are played in the tournament and, if no team has won the tournament at the end of four games, the tournament is declared a draw. What is the expected number of games in the tournament?

- (a) 4 (b) $11/4$ (c) $13/4$ (d) $19/4$ (e) $21/8$

19. A man starts with one dollar in a pot. A “play” consists of flipping a fair coin and,

- if heads occurs, doubling the amount in the pot,
- if tails occurs, losing one dollar from the pot. The game ends if the man has zero dollars or if he has played three times. Let Y denote the random variable which, for each outcome of the game, specifies the amount of money in the pot. What is the value of $\text{Var}(Y)$?

- (a) $9/8$ (b) $10/8$ (c) $12/8$ (d) $14/8$ (e) $447/64$

20. We are given an urn that has one red ball and one white ball. A fair die is thrown. If the number is a 1 or 2, one red ball is added to the urn. Otherwise two red balls are added to the urn. A ball is then drawn at random from the urn. Given that a red ball was drawn, what is the probability that a 1 or 2 appeared when the die was thrown?

- (a) $4/13$ (b) $5/13$ (c) $6/13$ (d) $7/13$ (e) $8/13$

21. In a certain college,

- 10 percent of the students are science majors.
- 10 percent are engineering majors.
- 80 percent are humanities majors.
- Of the science majors, 20 percent have read Newsweek.
- Of the engineering majors, 10 percent have read Newsweek.
- Of the humanities majors, 20 percent have read Newsweek.

Given that a student selected at random has read Newsweek, what is the probability that that student is an engineering major?

- (a) $1/19$ (b) $2/19$ (c) $5/19$ (d) $9/19$ (e) $10/19$

22. The probability of team A winning any game is $1/3$. Team A plays team B in a tournament. If either team wins two games in a row, that team is declared the winner. At most *four* games are played and, if no team has won the tournament at the end of four games, a draw is declared. Given that the tournament lasts more than two games, what is the probability that A is the winner?

- (a) $1/9$ (b) $2/9$ (c) $4/9$ (d) $5/9$ (e) $6/9$

23. Ten percent of the students are science majors (S), 20 percent are engineering majors (E), and 70 percent are humanities majors (H). Of S, 10 percent have read 2 or more articles in Newsweek, 20 percent 1 article, 70 percent 0 articles. For E, the corresponding percents are 5, 15, 80. For H they are 20, 30, 50. Given that a student has read 0 articles in Newsweek, what is the probability that the student is S or E (i.e., not H)?

- (a) $21/58$ (b) $23/58$ (c) $12/29$ (d) $13/29$ (e) $1/2$

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Answers: 1 (d), 2 (e), 3 (d), 4 (c), 5 (b), 6 (c), 7 (d), 8 (a), 9 (c), 10 (c), 11 (d), 12 (a), 13 (b), 14 (a), 15 (c), 16 (b), 17 (c), 18 (b), 19 (e), 20 (a), 21 (a), 22 (b), 23 (b).

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