

Multiple Choice Questions for Review

1. Suppose there are 12 students, among whom are three students, M , B , C (a Math Major, a Biology Major, a Computer Science Major). We want to send a delegation of four students (chosen from the 12 students) to a convention. How many ways can this be done so that the delegation includes *exactly* two (not more, not less) students from $\{M, B, C\}$?
(a) 32 (b) 64 (c) 88 (d) 108 (e) 144
2. The permutations of $\{a, b, c, d, e, f, g\}$ are listed in lex order. What permutations are just before and just after $bacdefg$?
(a) Before: $agfedbc$, After: $bacdfge$
(b) Before: $agfedcb$, After: $badcefg$
(c) Before: $agfebcd$, After: $bacedgf$
(d) Before: $agfedcb$, After: $bacdfge$
(e) Before: $agfedcb$, After: $bacdegf$
3. Teams A and B play in a basketball tournament. The first team to win two games in a row or a total of three games wins the tournament. What is the number of ways the tournament can occur?
(a) 8 (b) 9 (c) 10 (d) 11 (e) 12
4. The number of four letter words that can be formed from the letters in BUBBLE (each letter occurring at most as many times as it occurs in BUBBLE) is
(a) 72 (b) 74 (c) 76 (d) 78 (e) 80
5. The number of ways to seat 3 boys and 2 girls in a row if each boy must sit next to at least one girl is
(a) 36 (b) 48 (c) 148 (d) 184 (e) 248
6. Suppose there are ten balls in an urn, four blue, four red, and two green. The balls are also numbered 1 to 10. How many ways are there to select an ordered sample of four balls without replacement such that there are two blue balls and two red balls in the sample?
(a) 144 (b) 256 (c) 446 (d) 664 (e) 864
7. How many different rearrangements are there of the letters in the word BUBBLE?
(a) 40 (b) 50 (c) 70 (d) 80 (e) 120
8. The English alphabet has 26 letters of which 5 are vowels (A,E,I,O,U). How many seven letter words, with all letters distinct, can be formed that start with B, end with the letters ES, and have exactly three vowels? The “words” for this problem are just strings of letters and need not have linguistic meaning.
(a) $2^3 \times 3^4 \times 17$
(b) $2^3 \times 3^4 \times 19$

Basic Counting and Listing

- (c) $2^4 \times 3^4 \times 19$
(d) $2^4 \times 3^3 \times 19$
(e) $2^4 \times 3^3 \times 17$
9. The permutations on $\{a, b, c, d, e, f, g\}$ are listed in lex order. All permutations $x_1x_2x_3x_4x_5x_6x_7$ with $x_4 = a$ or $x_4 = c$ are kept. All others are discarded. In this reduced list what permutation is just after $dagcfbe$?
- (a) $dbacefg$
(b) $dbcaefg$
(c) $dbacgfe$
(d) $dagcfbe$
(e) $dcbaefg$
10. The number of four letter words that can be formed from the letters in SASSABY (each letter occurring at most as many times as it occurs in SASSABY) is
- (a) 78 (b) 90 (c) 108 (d) 114 (e) 120
11. How many different rearrangements are there of the letters in the word TATARS if the two A's are never adjacent?
- (a) 24 (b) 120 (c) 144 (d) 180 (e) 220
12. Suppose there are ten balls in an urn, four blue, four red, and two green. The balls are also numbered 1 to 10. How many ways are there to select an *ordered* sample of four balls without replacement such that the number $B \geq 0$ of blue balls, the number $R \geq 0$ of red balls, and the number $G \geq 0$ of green balls are all different?
- (a) 256 (b) 864 (c) 1152 (d) 1446 (e) 2144
13. Suppose there are ten balls in an urn, four blue, four red, and two green. The balls are also numbered 1 to 10. You are asked to select an *ordered* sample of four balls without replacement. Let $B \geq 0$ be the number of blue balls, $R \geq 0$ be the number of red balls, and $G \geq 0$ be the number of green balls in your sample. How many ways are there to select such a sample if *exactly* one of B , R , or G must be zero?
- (a) 256 (b) 1152 (c) 1446 (d) 2144 (e) 2304
14. The number of partitions of $X = \{a, b, c, d\}$ with a and b in the same block is
- (a) 4 (b) 5 (c) 6 (d) 7 (e) 8
15. Let W_{ab} and W_{ac} denote the set of partitions of $X = \{a, b, c, d, e\}$ with a and b belonging to the same block and with a and c belonging to the same block, respectively. Similarly, let W_{abc} denote the set of partitions of $X = \{a, b, c, d, e\}$ with a , b , and c belonging to the same block. What is $|W_{ab} \cup W_{ac}|$? (Note: $B(3) = 5$, $B(4) = 15$, $B(5) = 52$, where $B(n)$ is the number of partitions of an n -element set).
- (a) 25 (b) 30 (c) 35 (d) 40 (e) 45
16. The number of partitions of $X = \{a, b, c, d, e, f, g\}$ with a , b , and c in the same block and c , d , and e in the same block is

Review Questions

- (a) 2 (b) 5 (c) 10 (d) 15 (e) 52
17. Three boys and four girls sit in a row with all arrangements equally likely. Let x be the probability that no two boys sit next to each other. What is x ?
- (a) $1/7$ (b) $2/7$ (c) $3/7$ (d) $4/7$ (e) $5/7$
18. A man is dealt 4 spade cards from an ordinary deck of 52 cards. He is given 2 more cards. Let x be the probability that they both are the same suit. Which is true?
- (a) $.2 < x \leq .3$
(b) $0 < x \leq .1$
(c) $.1 < x \leq .2$
(d) $.3 < x \leq .4$
(e) $.4 < x \leq .5$
19. Six light bulbs are chosen at random from 15 bulbs of which 5 are defective. What is the probability that exactly one is defective?
- (a) $C(5, 1)C(10, 6)/C(15, 6)$
(b) $C(5, 1)C(10, 5)/C(15, 6)$
(c) $C(5, 1)C(10, 1)/C(15, 6)$
(d) $C(5, 0)C(10, 6)/C(15, 6)$
(e) $C(5, 0)C(10, 5)/C(15, 6)$
20. A small deck of five cards are numbered 1 to 5. First one card and then a second card are selected at random, with replacement. What is the probability that the sum of the values on the cards is a prime number?
- (a) $10/25$ (b) $11/25$ (c) $12/25$ (d) $13/25$ (e) $14/25$
21. Let A and B be events with $P(A) = 6/15$, $P(B) = 8/15$, and $P((A \cup B)^c) = 3/15$. What is $P(A \cap B)$?
- (a) $1/15$ (b) $2/15$ (c) $3/15$ (d) $4/15$ (e) $5/15$
22. Suppose the odds of A occurring are 1:2, the odds of B occurring are 5:4, and the odds of both A and B occurring are 1:8. The odds of $(A \cap B^c) \cup (B \cap A^c)$ occurring are
- (a) 2:3 (b) 4:3 (c) 5:3 (d) 6:3 (e) 7:3
23. A pair of fair dice is tossed. Find the probability that the greatest common divisor of the two numbers is one.
- (a) $12/36$ (b) $15/36$ (c) $17/36$ (d) $19/36$ (e) $23/36$
24. Three boys and three girls sit in a row. Find the probability that exactly two of the girls are sitting next to each other (the remaining girl separated from them by at least one boy).
- (a) $4/20$ (b) $6/20$ (c) $10/20$ (d) $12/20$ (e) $13/20$
25. A man is dealt 4 spade cards from an ordinary deck of 52 cards. If he is given five more, what is the probability that none of them are spades?

Basic Counting and Listing

(a) $\binom{39}{1}/\binom{48}{5}$ (b) $\binom{39}{2}/\binom{48}{5}$ (c) $\binom{39}{3}/\binom{48}{5}$ (d) $\binom{39}{5}/\binom{48}{5}$ (e) $\binom{39}{6}/\binom{48}{5}$

Answers: 1 (d), 2 (e), 3 (c), 4 (a), 5 (a), 6 (e), 7 (e), 8 (c), 9 (a), 10 (d), 11 (b), 12 (c), 13 (e), 14 (b), 15 (a), 16 (b), 17 (b), 18 (a), 19 (b), 20 (b), 21 (b), 22 (d), 23 (e), 24 (d), 25 (d).

Notation Index

- B_n (Bell numbers) CL-27
 $\binom{n}{k}$ (binomial coefficient) CL-15
 $\binom{n}{m_1, m_2, \dots}$ (multinomial coefficient) CL-20
 $C(n, k)$ (binomial coefficient) CL-15
 $(n)_k$ (falling factorial) CL-9
 \mathbb{N} (natural numbers) CL-13
 $\mathcal{P}_k(A)$ (k -subsets of A) CL-15
 \mathbb{R} (real numbers) CL-28
- Set notation
- $\sim A$ (complement) CL-14
 - \in and \notin (in and not in) CL-14
 - A' (complement) CL-14
 - $A - B$ (difference) CL-14
 - $A \cap B$ (intersection) CL-14
 - $A \cup B$ (union) CL-14
 - $A \setminus B$ (difference) CL-14
 - $A \subseteq B$ (subset) CL-14
 - $A \times B$ (Cartesian product) CL-4
 - A^c (complement) CL-14
 - $\mathcal{P}_k(A)$ (k -subsets of A) CL-15
 - $|A|$ (cardinality) CL-3, CL-14
- $S(n, k)$ (Stirling numbers) CL-25
 \mathbb{Z} (integers) CL-13

Subject Index

- Absorption rule CL-15
- Algebraic rules for sets CL-15
- Associative rule CL-15

- Bell numbers CL-27
- Binomial coefficients CL-15
 - recursion CL-23
- Binomial theorem CL-18
- Blocks of a partition CL-20, CL-25

- Card hands
 - and multinomial coefficients CL-23
 - full house CL-19
 - straight CL-26
 - two pairs CL-19
- Cardinality CL-3
- Cardinality of a set CL-14
- Cartesian product CL-4
- Commutative rule CL-15
- Composition of an integer CL-8

- DeMorgan's rule CL-15
- Dictionary order CL-4
- Direct (Cartesian) product CL-4
- Distribution
 - hypergeometric CL-32
 - uniform CL-28
- Distributive rule CL-15
- Double negation rule CL-15

- Elementary event CL-29
- Error
 - percentage CL-10
 - relative CL-10

- Event CL-28
 - elementary=simple CL-29

- Factorial
 - falling CL-9
- Factorial estimate (Stirling's formula) CL-10
- Falling factorial $(n)_k$ CL-9
- Function
 - generating CL-16

- Generating function CL-16
- Geometric probability CL-34

- Hypergeometric probability CL-32

- Idempotent rule CL-15
- Inclusion and exclusion CL-31, CL-39

- Lexicographic order (lex order) CL-4
- List CL-2
 - circular CL-10
 - with repetition CL-3
 - without repetition CL-3, CL-9

- Multinomial coefficient CL-20
- Multiset CL-3

- Numbers
 - Bell CL-27
 - binomial coefficients CL-15
 - Stirling (set partitions) CL-25