Multiple Choice Questions for Review

In each case there is one correct answer (given at the end of the problem set). Try to work the problem first without looking at the answer. Understand both why the correct answer is correct and why the other answers are wrong.

1. Which of the following sequences is described, as far as it goes, by an explicit formula ($n \geq 0$) of the form $g_n = \lfloor \frac{n}{k} \rfloor$?

(a) 0000111122222

(b) 001112223333

(c) 000111222333

(d) 0000011112222

(e) 0001122233444

2. Given that $k > 1$, which of the following sum or product representations is **WRONG**?

(a) $(2^2+1)(3^2+1)\cdots(k^2+1) = \prod_{j=2}^{k}[(j+1)^2 - 2j]$

(b) $(1^3-1)+(2^3-2)+\cdots+(k^3-k) = \sum_{j=1}^{k-1}[(k-j)^3-(k-j)]$

(c) $(1-r)(1-r^2)(1-r^3)\cdots(1-r^k) = \prod_{j=0}^{k-1}(1-r^{k-j})$

(d) $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\cdots+\frac{k-1}{k!} = \sum_{j=2}^{k}\frac{j-1}{j!}$

(e) $n+(n-1)+(n-2)+\cdots+(n-k) = \sum_{j=1}^{k+1}(n-j+1)$

3. Which of the following sums is gotten from $\sum_{i=1}^{n-1}\frac{i}{(n-i)^2}$ by the change of variable $j = i+1$?

(a) $\sum_{j=2}^{n}\frac{j-1}{(n-j+1)^2}$

(b) $\sum_{j=2}^{n}\frac{j-1}{(n-j-1)^2}$

(c) $\sum_{j=2}^{n}\frac{j}{(n-j+1)^2}$

(d) $\sum_{j=2}^{n}\frac{j}{(n-j-1)^2}$

(e) $\sum_{j=2}^{n}\frac{j+1}{(n-j+1)^2}$

4. We are going to prove by induction that $\sum_{i=1}^{n}Q(i) = n^2(n+1)$. For which choice of $Q(i)$ will induction work?

(a) $3i^2-2$ (b) $2i^2$ (c) $3i^3-i$ (d) $i(3i-1)$ (e) $3i^3-7i$

5. The sum $\sum_{k=1}^{n}(1+2+3+\cdots+k)$ is a polynomial in n of degree

(a) 3 (b) 1 (c) 2 (d) 4 (e) 5

6. We are going to prove by induction that for all integers $k \geq 1$,

$$\sqrt{k} \leq \frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\cdots+\frac{1}{\sqrt{k}}.$$

Clearly this is true for $k = 1$. Assume the Induction Hypothesis (IH) that $\sqrt{n} \le \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots \frac{1}{\sqrt{n}}$. Which is a correct way of concluding this proof by induction?

(a) By IH, $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots \frac{1}{\sqrt{n+1}} \ge \sqrt{n} + \frac{1}{\sqrt{n+1}} = \sqrt{n+1} + 1 \ge \sqrt{n+1}$.

(b) By IH, $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots \frac{1}{\sqrt{n+1}} \ge \sqrt{n+1} + \frac{1}{\sqrt{n+1}} \ge \sqrt{n+1}$.

(c) By IH, $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots \frac{1}{\sqrt{n+1}} \ge \sqrt{n} + 1 \ge \sqrt{n+1}$.

(d) By IH, $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots \frac{1}{\sqrt{n+1}} \ge \sqrt{n} + \frac{1}{\sqrt{n}} \ge \frac{\sqrt{n}\sqrt{n+1}}{\sqrt{n}} \ge \frac{n+1}{\sqrt{n+1}} = \sqrt{n+1}$.

(e) By IH, $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots \frac{1}{\sqrt{n+1}} \ge \sqrt{n} + \frac{1}{\sqrt{n+1}} = \frac{\sqrt{n}\sqrt{n+1}+1}{\sqrt{n+1}} \ge \frac{\sqrt{n}\sqrt{n+1}}{\sqrt{n+1}} = \frac{n+1}{\sqrt{n+1}} = \sqrt{n+1}$.

7. Suppose $b_1, b_2, b_3, \cdots$ is a sequence defined by $b_1 = 3$, $b_2 = 6$, $b_k = b_{k-2} + b_{k-1}$ for $k \ge 3$. Prove that b_n is divisible by 3 for all integers $n \ge 1$. Regarding the induction hypothesis, which is true?

(a) Assuming this statement is true for $k \le n$ is enough to show that it is true for $n+1$ and no weaker assumption will do since this proof is an example of "strong induction."

(b) Assuming this statement is true for n and $n-1$ is enough to show that it is true for $n+1$.

(c) Assuming this statement is true for n, $n-1$, and $n-3$ is enough to show that it is true for $n+1$ and no weaker assumption will do since you need three consecutive integers to insure divisibility by 3.

(d) Assuming this statement is true for n is enough to show that it is true for $n+1$.

(e) Assuming this statement is true for n and $n-3$ is enough to show that it is true for $n+1$ since 3 divides n if and only if 3 divides $n-3$.

8. Evaluate $\lim\limits_{n\to\infty} \dfrac{(-1)^{n^3} n^3 + 1}{2n^3 + 3}$.

(a) $-\infty$ (b) $+\infty$ (c) Does not exist. (d) $+1$ (e) -1

9. Evaluate $\lim\limits_{n\to\infty} \dfrac{\log_5(n)}{\log_9(n)}$.

(a) $\ln(9)/\ln(5)$ (b) $\ln(5)/\ln(9)$ (c) $5/9$ (d) $9/5$ (e) 0

10. Evaluate $\lim\limits_{n\to\infty} \dfrac{\cos(n)}{\log_2(n)}$.

(a) Does not exist. (b) 0 (c) $+1$ (d) -1 (e) $+\infty$

***11.** The series $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n\, n^{500}}{(1.0001)^n}$

(a) converges absolutely.

(b) converges conditionally, but not absolutely.

(c) converges to $+\infty$

(d) converges to $-\infty$

(e) is bounded but divergent.

***12.** The series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}\left(1+\frac{1}{n^2}\right)$.

(a) is bounded but divergent.

(b) converges absolutely.

(c) converges to $+\infty$

(d) converges to $-\infty$

(e) converges conditionally, but not absolutely.

Answers: **1** (c), **2** (b), **3** (a), **4** (d), **5** (a), **6** (e), **7** (b), **8** (c), **9** (a), **10** (b), **11** (a), **12** (e).

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