

Multiple Choice Questions for Review

In each case there is one correct answer (given at the end of the problem set). Try to work the problem first without looking at the answer. Understand both why the correct answer is correct and why the other answers are wrong.

- Which of the following statements is **FALSE**?
 - $2 \in A \cup B$ implies that if $2 \notin A$ then $2 \in B$.
 - $\{2, 3\} \subseteq A$ implies that $2 \in A$ and $3 \in A$.
 - $A \cap B \supseteq \{2, 3\}$ implies that $\{2, 3\} \subseteq A$ and $\{2, 3\} \subseteq B$.
 - $A - B \supseteq \{3\}$ and $\{2\} \subseteq B$ implies that $\{2, 3\} \subseteq A \cup B$.
 - $\{2\} \in A$ and $\{3\} \in A$ implies that $\{2, 3\} \subseteq A$.
- Let $A = \{0, 1\} \times \{0, 1\}$ and $B = \{a, b, c\}$. Suppose A is listed in lexicographic order based on $0 < 1$ and B is in alphabetic order. If $A \times B \times A$ is listed in lexicographic order, then the next element after $((1, 0), c, (1, 1))$ is
 - $((1, 0), a, (0, 0))$
 - $((1, 1), c, (0, 0))$
 - $((1, 1), a, (0, 0))$
 - $((1, 1), a, (1, 1))$
 - $((1, 1), b, (1, 1))$
- Which of the following statements is **TRUE**?
 - For all sets A , B , and C , $A - (B - C) = (A - B) - C$.
 - For all sets A , B , and C , $(A - B) \cap (C - B) = (A \cap C) - B$.
 - For all sets A , B , and C , $(A - B) \cap (C - B) = A - (B \cup C)$.
 - For all sets A , B , and C , if $A \cap C = B \cap C$ then $A = B$.
 - For all sets A , B , and C , if $A \cup C = B \cup C$ then $A = B$.
- Which of the following statements is **FALSE**?
 - $C - (B \cup A) = (C - B) - A$
 - $A - (C \cup B) = (A - B) - C$
 - $B - (A \cup C) = (B - C) - A$
 - $A - (B \cup C) = (B - C) - A$
 - $A - (B \cup C) = (A - C) - B$
- Consider the true theorem, "For all sets A and B , if $A \subseteq B$ then $A \cap B^c = \emptyset$." Which of the following statements is **NOT** equivalent to this statement:
 - For all sets A^c and B , if $A \subseteq B$ then $A^c \cap B^c = \emptyset$.
 - For all sets A and B , if $A^c \subseteq B$ then $A^c \cap B^c = \emptyset$.

Sets and Functions

- (c) For all sets A^c and B^c , if $A \subseteq B^c$ then $A \cap B = \emptyset$.
- (d) For all sets A^c and B^c , if $A^c \subseteq B^c$ then $A^c \cap B = \emptyset$.
- (e) For all sets A and B , if $A^c \supseteq B$ then $A \cap B = \emptyset$.
6. The power set $\mathcal{P}((A \times B) \cup (B \times A))$ has the same number of elements as the power set $\mathcal{P}((A \times B) \cup (A \times B))$ if and only if
- (a) $A = B$
- (b) $A = \emptyset$ or $B = \emptyset$
- (c) $B = \emptyset$ or $A = B$
- (d) $A = \emptyset$ or $B = \emptyset$ or $A = B$
- (e) $A = \emptyset$ or $B = \emptyset$ or $A \cap B = \emptyset$
7. Let $\sigma = 452631$ be a permutation on $\{1, 2, 3, 4, 5, 6\}$ in one-line notation (based on the usual order on integers). Which of the following is **NOT** a correct cycle notation for σ ?
- (a) $(614)(532)$
- (b) $(461)(352)$
- (c) $(253)(146)$
- (d) $(325)(614)$
- (e) $(614)(253)$
8. Let $f : X \rightarrow Y$. Consider the statement, “For all subsets C and D of Y , $f^{-1}(C \cap D^c) = f^{-1}(C) \cap [f^{-1}(D)]^c$. This statement is
- (a) True and equivalent to:
For all subsets C and D of Y , $f^{-1}(C - D) = f^{-1}(C) - f^{-1}(D)$.
- (b) False and equivalent to:
For all subsets C and D of Y , $f^{-1}(C - D) = f^{-1}(C) - f^{-1}(D)$.
- (c) True and equivalent to:
For all subsets C and D of Y , $f^{-1}(C - D) = f^{-1}(C) - [f^{-1}(D)]^c$.
- (d) False and equivalent to:
For all subsets C and D of Y , $f^{-1}(C - D) = f^{-1}(C) - [f^{-1}(D)]^c$.
- (e) True and equivalent to:
For all subsets C and D of Y , $f^{-1}(C - D) = [f^{-1}(C)]^c - f^{-1}(D)$.
9. Define $f(n) = \frac{n}{2} + \frac{1-(-1)^n}{4}$ for all $n \in \mathbb{Z}$. Thus, $f : \mathbb{Z} \rightarrow \mathbb{Z}$, \mathbb{Z} the set of all integers. Which is correct?
- (a) f is not a function from $\mathbb{Z} \rightarrow \mathbb{Z}$ because $\frac{n}{2} \notin \mathbb{Z}$.
- (b) f is a function and is onto and one-to-one.
- (c) f is a function and is not onto but is one-to-one.
- (d) f is a function and is not onto and not one-to-one

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- (e) f is a function and is onto but not one-to-one.
10. The number of partitions of $\{1, 2, 3, 4, 5\}$ into three blocks is $S(5, 3) = 25$. The total number of functions $f : \{1, 2, 3, 4, 5\} \rightarrow \{1, 2, 3, 4\}$ with $|\text{Image}(f)| = 3$ is
- 4×6
 - 4×25
 - 25×6
 - $4 \times 25 \times 6$
 - $3 \times 25 \times 6$
11. Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$. Let $h = g \circ f : X \rightarrow Z$. Suppose g is one-to-one and onto. Which of the following is **FALSE**?
- If f is one-to-one then h is one-to-one and onto.
 - If f is not onto then h is not onto.
 - If f is not one-to-one then h is not one-to-one.
 - If f is one-to-one then h is one-to-one.
 - If f is onto then h is onto.
12. Which of the following statements is **FALSE**?
- $\{2, 3, 4\} \subseteq A$ implies that $2 \in A$ and $\{3, 4\} \subseteq A$.
 - $\{2, 3, 4\} \in A$ and $\{2, 3\} \in B$ implies that $\{4\} \subseteq A - B$.
 - $A \cap B \supseteq \{2, 3, 4\}$ implies that $\{2, 3, 4\} \subseteq A$ and $\{2, 3, 4\} \subseteq B$.
 - $A - B \supseteq \{3, 4\}$ and $\{1, 2\} \subseteq B$ implies that $\{1, 2, 3, 4\} \subseteq A \cup B$.
 - $\{2, 3\} \subseteq A \cup B$ implies that if $\{2, 3\} \cap A = \emptyset$ then $\{2, 3\} \subseteq B$.
13. Let $A = \{0, 1\} \times \{0, 1\} \times \{0, 1\}$ and $B = \{a, b, c\} \times \{a, b, c\} \times \{a, b, c\}$. Suppose A is listed in lexicographic order based on $0 < 1$ and B is listed in lexicographic order based on $a < b < c$. If $A \times B \times A$ is listed in lexicographic order, then the next element after $((0, 1, 1), (c, c, c), (1, 1, 1))$ is
- $((1, 0, 1), (a, a, b), (0, 0, 0))$
 - $((1, 0, 0), (b, a, a), (0, 0, 0))$
 - $((1, 0, 0), (a, a, a), (0, 0, 1))$
 - $((1, 0, 0), (a, a, a), (1, 0, 0))$
 - $((1, 0, 0), (a, a, a), (0, 0, 0))$
14. Consider the true theorem, "For all sets A , B , and C if $A \subseteq B \subseteq C$ then $C^c \subseteq B^c \subseteq A^c$." Which of the following statements is **NOT** equivalent to this statement:
- For all sets A^c , B^c , and C^c , if $A^c \subseteq B^c \subseteq C^c$ then $C \subseteq B \subseteq A$.
 - For all sets A^c , B , and C^c , if $A^c \subseteq B \subseteq C^c$ then $C \subseteq B^c \subseteq A$.
 - For all sets A , B , and C^c , if $A^c \subseteq B \subseteq C$ then $C^c \subseteq B^c \subseteq A$.

Sets and Functions

- (d) For all sets A^c , B , and C^c , if $A^c \subseteq B^c \subseteq C$ then $C^c \subseteq B^c \subseteq A$.
- (e) For all sets A^c , B^c , and C^c , if $A^c \subseteq B^c \subseteq C$ then $C^c \subseteq B \subseteq A$.
15. Let $\mathcal{P}(A)$ denote the power set of A . If $\mathcal{P}(A) \subseteq B$ then
- (a) $2^{|A|} \leq |B|$
 - (b) $2^{|A|} \geq |B|$
 - (c) $2|A| < |B|$
 - (d) $|A| + 2 \leq |B|$
 - (e) $2^{|A|} \geq 2^{|B|}$
16. Let $f : \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \rightarrow \{a, b, c, d, e\}$. In one-line notation, $f = (e, a, b, b, a, c, c, a, c)$ (use number order on the domain). Which is correct?
- (a) $\text{Image}(f) = \{a, b, c, d, e\}$, $\text{Coimage}(f) = \{\{6, 7, 9\}, \{2, 5, 8\}, \{3, 4\}, \{1\}\}$
 - (b) $\text{Image}(f) = \{a, b, c, e\}$, $\text{Coimage}(f) = \{\{6, 7, 9\}, \{2, 5, 8\}, \{3, 4\}\}$
 - (c) $\text{Image}(f) = \{a, b, c, e\}$, $\text{Coimage}(f) = \{\{6, 7, 9\}, \{2, 5, 8\}, \{3, 4\}, \{1\}\}$
 - (d) $\text{Image}(f) = \{a, b, c, e\}$, $\text{Coimage}(f) = \{\{6, 7, 9, 2, 5, 8\}, \{3, 4\}, \{1\}\}$
 - (e) $\text{Image}(f) = \{a, b, c, d, e\}$, $\text{Coimage}(f) = \{\{1\}, \{3, 4\}, \{2, 5, 8\}, \{6, 7, 9\}\}$
17. Let $\Sigma = \{x, y\}$ be an alphabet. The strings of length seven over Σ are listed in dictionary (lex) order. What is the first string after $xxxxyx$ that is a palindrome (same read forwards and backwards)?
- (a) $xxxxxy$
 - (b) $xxxyxx$
 - (c) $xyxyxx$
 - (d) $xyyyxx$
 - (e) $yxxyyx$
18. Let $\sigma = 681235947$ and $\tau = 627184593$ be permutations on $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ in one-line notation (based on the usual order on integers). Which of the following is a correct cycle notation for $\tau \circ \sigma$?
- (a) (124957368)
 - (b) (142597368)
 - (c) (142953768)
 - (d) (142957368)
 - (e) (142957386)

Answers: 1 (e), 2 (c), 3 (b), 4 (d), 5 (a), 6 (d), 7 (b), 8 (a), 9 (e), 10 (d), 11 (a), 12 (b), 13 (e), 14 (d), 15 (a), 16 (c), 17 (b), 18 (d).

Notation Index

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