Multiple Choice Questions for Review

In each case there is one correct answer (given at the end of the problem set). Try to work the problem first without looking at the answer. Understand both why the correct answer is correct and why the other answers are wrong.

1. Consider the statement form $p \Rightarrow q$ where $p =$"If Tom is Jane's father then Jane is Bill's niece" and $q =$"Bill is Tom's brother." Which of the following statements is equivalent to this statement?

(a) If Bill is Tom's Brother, then Tom is Jane's father and Jane is not Bill's niece.

(b) If Bill is not Tom's Brother, then Tom is Jane's father and Jane is not Bill's niece.

(c) If Bill is not Tom's Brother, then Tom is Jane's father or Jane is Bill's niece.

(d) If Bill is Tom's Brother, then Tom is Jane's father and Jane is Bill's niece.

(e) If Bill is not Tom's Brother, then Tom is not Jane's father and Jane is Bill's niece.

2. Consider the statement, "If n is divisible by 30 then n is divisible by 2 and by 3 and by 5." Which of the following statements is equivalent to this statement?

(a) If n is not divisible by 30 then n is divisible by 2 or divisible by 3 or divisible by 5.

(b) If n is not divisible by 30 then n is not divisible by 2 or not divisible by 3 or not divisible by 5.

(c) If n is divisible by 2 and divisible by 3 and divisible by 5 then n is divisible by 30.

(d) If n is not divisible by 2 or not divisible by 3 or not divisible by 5 then n is not divisible by 30.

(e) If n is divisible by 2 or divisible by 3 or divisible by 5 then n is divisible by 30.

3. Which of the following statements is the contrapositive of the statement, "You win the game if you know the rules but are not overconfident."

(a) If you lose the game then you don't know the rules or you are overconfident.

(b) A sufficient condition that you win the game is that you know the rules or you are not overconfident.

(c) If you don't know the rules or are overconfident you lose the game.

(d) If you know the rules and are overconfident then you win the game.

(e) A necessary condition that you know the rules or you are not overconfident is that you win the game.

4. The statement form $(p \Leftrightarrow r) \Rightarrow (q \Leftrightarrow r)$ is equivalent to

(a) $[(\sim p \vee r) \wedge (p \vee \sim r)] \vee \sim[(\sim q \vee r) \wedge (q \vee \sim r)]$

(b) $\sim[(\sim p \vee r) \wedge (p \vee \sim r)] \wedge [(\sim q \vee r) \wedge (q \vee \sim r)]$

(c) $[(\sim p \vee r) \wedge (p \vee \sim r)] \wedge [(\sim q \vee r) \wedge (q \vee \sim r)]$

(d) $[(\sim p \vee r) \wedge (p \vee \sim r)] \vee [(\sim q \vee r) \wedge (q \vee \sim r)]$

(e) $\sim[(\sim p \vee r) \wedge (p \vee \sim r)] \vee [(\sim q \vee r) \wedge (q \vee \sim r)]$

5. Consider the statement, “Given that people who are in need of refuge and consolation are apt to do odd things, it is clear that people who are apt to do odd things are in need of refuge and consolation.” This statement, of the form $(P \Rightarrow Q) \Rightarrow (Q \Rightarrow P)$, is logically equivalent to

(a) People who are in need of refuge and consolation are not apt to do odd things.

(b) People are apt to do odd things if and only if they are in need of refuge and consolation.

(c) People who are apt to do odd things are in need of refuge and consolation.

(d) People who are in need of refuge and consolation are apt to do odd things.

(e) People who aren’t apt to do odd things are not in need of refuge and consolation.

6. A sufficient condition that a triangle T be a right triangle is that $a^2 + b^2 = c^2$. An equivalent statement is

(a) If T is a right triangle then $a^2 + b^2 = c^2$.

(b) If $a^2 + b^2 = c^2$ then T is a right triangle.

(c) If $a^2 + b^2 \neq c^2$ then T is not a right triangle.

(d) T is a right triangle only if $a^2 + b^2 = c^2$.

(e) T is a right triangle unless $a^2 + b^2 = c^2$.

7. Which of the following statements is **NOT** equivalent to the statement, “There exists either a computer scientist or a mathematician who knows both discrete math and Java.”

(a) There exists a person who is a computer scientist and who knows both discrete math and Java or there exists a person who is a mathematician and who knows both discrete math and Java.

(b) There exists a person who is a computer scientist or there exists a person who is a mathematician who knows discrete math or who knows Java.

(c) There exists a person who is a computer scientist and who knows both discrete math and Java or there exists a mathematician who knows both discrete math and Java.

(d) There exists a computer scientist who knows both discrete math and Java or there exists a person who is a mathematician who knows both discrete math and Java.

(e) There exists a person who is a computer scientist or a mathematician who knows both discrete math and Java.

8. Which of the following is the negation of the statement, “For all odd primes $p < q$ there exists positive non-primes $r < s$ such that $p^2 + q^2 = r^2 + s^2$.”

(a) For all odd primes $p < q$ there exists positive non-primes $r < s$ such that $p^2+q^2 \neq r^2 + s^2$.

(b) There exists odd primes $p < q$ such that for all positive non-primes $r < s$, $p^2+q^2 = r^2 + s^2$.

(c) There exists odd primes $p < q$ such that for all positive non-primes $r < s$, $p^2+q^2 \neq r^2+s^2$.

(d) For all odd primes $p < q$ and for all positive non-primes $r < s$, $p^2+q^2 \neq r^2+s^2$.

(e) There exists odd primes $p < q$ and there exists positive non-primes $r < s$ such that $p^2+q^2 \neq r^2+s^2$

9. Consider the following assertion: "The two statements
(1) $\exists x \in D, (P(x) \wedge Q(x))$ and
(2) $(\exists x \in D, P(x)) \wedge (\exists x \in D, Q(x))$ have the same
truth value." Which of the following is correct?

(a) This assertion is false. A counterexample is $D = \mathbb{N}$, $P(x) =$ "x is divisible by 6," $Q(x) =$ "x is divisible by 3."

(b) This assertion is true. The proof follows from the distributive law for $\wedge$.

(c) This assertion is false. A counterexample is $D = \mathbb{Z}$, $P(x) =$ "$x < 0$," $Q(x) =$ "$x \geq 0$."

(d) This assertion is true. To see why, let $D = \mathbb{N}$, $P(x) =$ "x is divisible by 6," $Q(x) =$ "x is divisible by 3." If $x = 6$, then x is divisible by both 3 and 6 so both statements in the assertion have the same truth value for this x.

(e) This assertion is false. A counterexample is $D = \mathbb{N}$, $P(x) =$ "x is a square," $Q(x) =$ "x is odd."

10. Which of the following is an unsolved conjecture?

(a) $\exists n \in \mathbb{N},\ 2^{2^n}+1 \notin \mathbb{P}$

(b) $\exists\, K \in \mathbb{N},\ \forall n \geq K$, n odd, $\exists\, p,q,r \in \mathbb{P},\ n = p+q+r$

(c) $(\exists\, x,y,z,n \in \mathbb{N}^+,\ x^n + y^n = z^n) \;\Leftrightarrow\; (n = 1,2)$

(d) $\forall\, m \in \mathbb{N}\,,\ \exists\, n \geq m$, n even, $\exists\, p,q \in \mathbb{P},\ n = p+q$

(e) $\forall\, m \in \mathbb{N}\,,\ \exists\, n \geq m,\ n \in \mathbb{P}$ and $n+2 \in \mathbb{P}$

11. Which of the following is a solved conjecture?

(a) $\forall\, m \in \mathbb{N}\,,\ \exists\, n \geq m$, n odd, $\exists\, p,q \in \mathbb{P},\ n = p+q$

(b) $\forall\, m \in \mathbb{N}\,,\ \exists\, n \geq m,\ n \in \mathbb{P}$ and $n+2 \in \mathbb{P}$

(c) $\forall\, m \in \mathbb{N}\,,\ \exists\, n \geq m,\ 2^{2^n}+1 \in \mathbb{P}$

(d) $\forall k \in \mathbb{N}\,, \exists\, p \in \mathbb{P},\ p \geq k\,, 2^p - 1 \in \mathbb{P}$

(e) $\forall\, n \geq 4,\ n$ even , $\exists\, p,q \in \mathbb{P},\ n = p+q$

Answers: **1** (b), **2** (d), **3** (a), **4** (e), **5** (c), **6** (b), **7** (b), **8** (c), **9** (c), **10** (e), **11** (a).

Notation Index

Logic notation
- $\exists$ (for some) Lo-13
- $\forall$ (for all) Lo-12
- $\sim$ (not) Lo-2
- $\wedge$ (and) Lo-2
- $\Leftrightarrow$ (if and only if) Lo-6
- $\vee$ (or) Lo-2
- $\Rightarrow$ (if . . . then) Lo-5

$\mathbb{N}$ (Natural numbers) Lo-13

$\mathbb{P}$ (Prime numbers) Lo-13

$\mathbb{R}$ (Real numbers) Lo-13

Sets of numbers
- $\mathbb{N}$ (Natural numbers) Lo-13
- $\mathbb{P}$ (Prime numbers) Lo-13
- $\mathbb{R}$ (Real numbers) Lo-13
- $\mathbb{Z}$ (Integers) Lo-13

$\mathbb{Z}$ (Integers) Lo-13

Subject Index

Absorption rule Lo-3
Algebraic rules for
  predicate logic Lo-19
  statement forms Lo-3
Associative rule Lo-3

Biconditional (= if and only if) Lo-6
Bound rule Lo-3

Commutative rule Lo-3
Composite number Lo-13
Conditional (= if ... then) Lo-5
Conjecture
  Goldbach's Lo-13
  Twin Prime Lo-16
Contradiction Lo-2
Contrapositive Lo-6
Converse Lo-6

DeMorgan's rule Lo-3
Distributive rule Lo-3
Double implication (= if and only if) Lo-6
Double negation rule Lo-3

English to logic
  "for all" Lo-12
  "for some" Lo-13
  "if and only if" Lo-7
  method for implication Lo-8
  "necessary" Lo-7
  "only if" Lo-7
  "requires" Lo-8
  "sufficient" Lo-7
  "there exists" Lo-13
  "unless" Lo-8

Existential quantifier ($\exists$) Lo-13

Fermat number Lo-16
Fermat's Last Theorem Lo-18
For all (logic: $\forall$) Lo-12
For some (logic: $\exists$) Lo-13

Goldbach's conjecture Lo-13

Idempotent rule Lo-3
If ... then Lo-5
If and only if (logic) Lo-7
Implication Lo-5
Inverse Lo-6

Logic
  predicate Lo-12
  propositional Lo-1

Mersenne number Lo-17

Necessary (logic) Lo-7
Negation rule Lo-3
Number
  composite Lo-13
  Fermat: F_n Lo-16
  integer: $\mathbb{Z}$ Lo-13
  Mersenne: M_p Lo-17
  natural: $\mathbb{N}$ Lo-13
  perfect Lo-17
  prime Lo-13
  prime: $\mathbb{P}$ Lo-13
  real: $\mathbb{R}$ Lo-13

Index

Number theory
- elementary Lo-13

Only if (logic) Lo-7

Perfect
- number Lo-17

Predicate logic
- algebraic rules Lo-19
- predicate Lo-12
- quantifier Lo-12
- truth set Lo-12

Prime number Lo-13

Propositional logic Lo-1
- algebraic rules Lo-3

Quantifier
- existential (∃) Lo-13
- negation of Lo-15
- universal (∀) Lo-12

Rule
- absorption Lo-3
- associative Lo-3
- bound Lo-3
- commutative Lo-3
- DeMorgan's Lo-3
- distributive Lo-3
- double negation Lo-3
- idempotent Lo-3
- negation Lo-3

Set
- as a predicate Lo-14

Statement form Lo-1
- Boolean function and Lo-8

Statement variable Lo-1

Sufficient (logic) Lo-7

Tautology Lo-2

There exists (logic: ∃) Lo-13

Truth set (predicate logic) Lo-12

Truth table Lo-2

Twin Prime conjecture Lo-16

Universal quantifier (∀) Lo-12

Unless (logic) Lo-8