Final Exam Sneak Preview

Begin by filling in the following using UPPER CASE letters.

First Name: ______________________________

Last Name: ______________________________

You may refer, during this exam, only to the following course notes: Computability Crib Sheet, Example of an NP-completeness proof, Notes on Randomized Algorithms and Decision versus Search. The copies of these you bring to the exam should not be annotated. No other material is permitted. In particular you may not refer to your course notes, the textbook, homeworks, your own homeworks, or their posted solutions. Electronic devices (computers, calculators, cell phones, iPods, whatever) are not permitted either.

At the end of the exam you will find scratch paper. Use this to develop your solutions. Write your final answers on the exam sheet itself, in the assigned space below the problem, or on the extra blank pages that follow.

Write clearly, correctly and concisely. You will lose points for writing things that do not make mathematical sense. Write left to right, top to bottom; don’t scatter information all over the page. Write a single answer, not many; if you write many different things you will not get credit if one of them is right and others are wrong. Developing your solutions first on the scratch paper will improve the quality of your actual answers.

You may use without proof the NP-completeness of the following problems but no others:

SAT, 3-SAT, INDEPENDENT-SET, CLIQUE, VERTEX-COVER, 3-COLORING, SUBSET-SUM, INTEGER-PROGRAMMING, SET-COVER.

Additionally, you may use without proof only results from class or homeworks. If you are uncertain about whether or not something can be used without proof, ask the proctor.

Exams not returned within two minutes of the announcement of the end of the examination period will not be accepted.

We fix the alphabet \( \Sigma = \{0, 1\} \). A string is a member of \( \Sigma^* \). A language is a subset of \( \Sigma^* \). If \( w \) is a string then \( |w| \) denotes its length, and if \( S \) is a set then \( |S| \) denotes its size.

Good luck!
**Scores:** To be filled in at grading time.

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Rules for Sneak Preview

The following contains snippets of the problems to appear on your exam that you can use to direct your study. To take advantage of this you must agree to the following rules. If you are not willing to abide by these rules, stop now.

In researching or studying for the questions that follow, you must work entirely on your own. You may not consult other students in the class, the instructor, TA or people outside the class.

You may use only course materials, meaning your classnotes, the content of the course webpage and the textbook. You may NOT use the Internet. Do NOT search for terms you find in the following. Do not use any other books or materials.
Problem 1 [12 points] Briefly explain what are the following theses and how we gain confidence in them.
Problem 2 [20 points] A randomized TM $M$, given an input $x$ and having some string $R$ placed on its random tape, can not only accept or reject but output some string to its output tape. We denote this output by $M(x; R)$. 
Problem 3 [20 points] A bipartite graph $G = (L, R, E)$ has two disjoint sets of vertices: $L$, the set of “left-hand-side” vertices, and $R$, the set of “right-hand-side” vertices. Edges of the graph must cross
Problem 4 [20 points] We say that TM \( V \) is an \( \textbf{NP} \)-verifier if it runs in time polynomial in the length of its first input. To any such verifier we associate the language

\[
L_V = \{ x \in \Sigma^* : \exists y \text{ such that } V(x, y) \text{ accepts} \}.
\]

A TM \( M_V \) solves the search problem for \( V \) if
**Problem 5 [15 points]** A language $L$ is in the complexity class
**Problem 6 [13 points]** A verifier $V$ is said to define an interactive proof for language $L$ if there exists a prover $P$ and a constant $\epsilon < 1/2$ such that for all $x \in \Sigma^*$ the following are true:

1. **Completeness:** If $x \in L$ then $\text{AccPr}_V^P(x) \geq 1 - \epsilon$
2. **Soundness:** If $x \notin L$ then $\text{AccPr}_V^\hat{P}(x) \leq \epsilon$ for all $\hat{P}$.
[Scratch Paper. Use to develop solutions before writing answers. This will not be looked at by graders.]
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