CSE 101 Homework 2

Spring 2017

This homework is due on gradescope Friday April 28th at 11:59pm. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in \LaTeX is recommend though not required.

Recommended practice problems: Chapter 4, problems 1, 5, 8, 9, 17, 19

**Question 1** (Priority Queue Tradeoffs, 30 points) Suppose that Dijkstra’s algorithm is being run on a graph $G = (V,E)$ with the priority queue implemented using a $d$-ary heap. By adjusting $d$, we can try to achieve better runtimes for the algorithm.

(a) Show that if $|E| > |V|^{11/10}$, that $d$ can be selected so that Dijkstra’s algorithm runs in linear time. [10 points]

(b) What is the runtime if $d$ is selected to be $|E|/|V|$ (assuming that this is an integer at least 2)? Show that no other choice of $d$ improves on this runtime by more than a constant multiple. [10 points]

(c) For a fixed value of $|V|$, what is the biggest possible ratio between this runtime and the runtime of Dijkstra using a Fibonacci heap? [10 points]

**Question 2** (Vertex labelling, 30 points) Suppose that you are given a directed graph $G = (V,E)$ and a function $f : V \to \mathbb{R}$. We would like to compute another function $g : V \to \mathbb{R}$ so that for every vertex $v \in V$ we have that

$$g(v) + \sum_{(v,w) \in E} g(w) = f(v).$$

(a) Show that if $G$ is a DAG that there is a linear time algorithm for computing such a $g$. Hint: consider determining the values $g(v)$ one at a time in some particular order. [20 points]

(b) Given an example to show that this is not always even possible if $G$ is not a DAG. [10 points]

**Question 3** (Shopping Trip, 40 points) Jade lives in a city whose roads are described by a directed graph $G$. She wants to go on a shopping trip where she visits every vertex in some subset $S$ of $V$. She wants to find an efficient algorithm to determine whether or not it is possible to do this.

(a) If $G$ is a DAG, use a topological sort to figure out what order the vertices of $S$ must be visited in. [10 points]

(b) Use the above to find a linear time algorithm to solve Jade’s problem when $G$ is a DAG. Hint: you might need to remove edges from $G$ that skip stores that she needs to visit. [15 points]

(c) Find a linear time algorithm to solve Jade’s problem in an arbitrary directed graph, $G$. Hint: first find the strongly connected components and reduce to the case of a DAG. [15 points]

**Question 4** (Extra credit, 1 point) Approximately how much time did you spend working on this homework?