1 NCC / NSSD Equivalence Proof

**Theorem 1.** Maximizing the normalized cross correlation (NCC) is equivalent to minimizing the normalized sum of squared differences (NSSD).

**Proof.** Let $\mathbf{w}_1$ and $\mathbf{w}_2$ be two equal-length vectors and define $\tilde{\mathbf{w}}_1 = \frac{\mathbf{w}_1 - \mathbf{w}}{\sqrt{(\mathbf{w}_1 - \mathbf{w})(\mathbf{w}_1 - \mathbf{w})^T}}$ and $\tilde{\mathbf{w}}_2 = \frac{\mathbf{w}_2 - \mathbf{w}}{\sqrt{(\mathbf{w}_2 - \mathbf{w})(\mathbf{w}_2 - \mathbf{w})^T}}$.

The NCC cost metric is $c_{NCC} = \tilde{\mathbf{w}}_1^T \tilde{\mathbf{w}}_2$. We now relate $c_{NSSD}$ to $c_{NCC}$,

$$c_{NSSD} = (\tilde{\mathbf{w}}_1 - \tilde{\mathbf{w}}_2)^T (\tilde{\mathbf{w}}_1 - \tilde{\mathbf{w}}_2)$$

$$= \tilde{\mathbf{w}}_1^T \tilde{\mathbf{w}}_1 + \tilde{\mathbf{w}}_2^T \tilde{\mathbf{w}}_2 - 2 \tilde{\mathbf{w}}_1^T \tilde{\mathbf{w}}_2$$

$$= 2 - 2 \tilde{\mathbf{w}}_1^T \tilde{\mathbf{w}}_2$$ (since $\tilde{\mathbf{w}}_1^T \tilde{\mathbf{w}}_1 = \tilde{\mathbf{w}}_2^T \tilde{\mathbf{w}}_2 = 1$)

$$= 2 - 2c_{NCC}.$$ (4)

Since the derivative of $c_{NSSD}$ is equal to $-2$ times the derivative of $c_{NCC}$ (with respect to any variable), this implies that maxima of $c_{NCC}$ occur at minima of $c_{NSSD}$. □