Problem 1

Prove that Minkowski’s constant $\gamma_n$ satisfies $\gamma_n = \Theta(n)$. What are the best constants $c_1, c_2$ for which you can prove $c_1 n \leq \gamma_n \leq c_2 n$?

Problem 2

Show that if $A \in \mathbb{Z}_q^{k \times n}$ is chosen uniformly at random, then $\Pr\{\det(\Lambda_q^\perp(A)) = q^k\} = \Pr\{\det(\Lambda_q(A)) = q^{n-k}\} = \Pr\{AZ_q^n = \mathbb{Z}_q^k\} \geq 1 - 1/q^{n-k}$.

Problem 3

Prove that there is a constant $\delta > 0$ such that if $\Lambda$ is chosen according to $\Lambda_q^\perp(n,k)$, then $\Pr\{\lambda(\Lambda) < \delta \sqrt{n}q^{k/n}\} \leq 1/2^n$. [Hint: consider all integer vectors of norm at most $\delta \sqrt{n}q^{k/n}$ and use a union bound.]

Problem 4

Prove that if $\Lambda$ is chosen according to $\Lambda_q^\perp(n,k)$, then $\Pr\{\frac{1}{2} \cdot \sqrt{n} \cdot q^{k/n} \leq 2\mu(\Lambda)\} \leq 1/2^n$. [Hint: Prove the bound for $\Lambda_q(n,n-k) \approx \Lambda_q^\perp(n,k)$ instead, and use duality and the transference theorems.]

Problem 5

Prove that there exists an $n$-dimensional lattice $\Lambda$ such that $\lambda_1(\Lambda) \cdot \lambda_1(\Lambda) \geq \Omega(n)$. 
