Sampling and Reconstruction of Visual Appearance
CSE 274 [Winter 2018], Lecture 2
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Motivation: BRDFs, Radiometry
- Basics of Illumination, Reflection
- Formal radiometric analysis (not ad-hoc)
- Reflection Equation
- Monte Carlo Rendering next week
- Appreciate formal analysis in a graduate course, even if not absolutely essential in practice

Motivation: BRDFs, Radiometry

Radiometry
- Physical measurement of electromagnetic energy
- Measure spatial (and angular) properties of light
  - Radiance, Irradiance
  - Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF
  - Reflection Equation
  - Simple BRDF models

Light
Visible electromagnetic radiation
Power spectrum
Polarization
Photon (quantum effects)
Wave (interference, diffraction)

Radiometry
- Physical measurement of electromagnetic energy

Angles and Solid Angles
- Angle \( \theta = \frac{L}{r} \)
  \( \Rightarrow \) circle has 2\( \pi \) radians
- Solid angle \( \Omega = \frac{A}{R^2} \)
  \( \Rightarrow \) sphere has 4\( \pi \) steradians

Differential Solid Angles
\[
d\Omega = (r \, d\theta)(r \sin \theta \, d\phi)
= r^2 \sin \theta \, d\theta \, d\phi
\]
Radiance

- Power per unit projected area perpendicular to the ray per unit solid angle in the direction of the ray.
- Symbol: \( L(x, \omega) \) (W/m\(^2\) sr)
- Flux given by \( d\Phi = L(x, \omega) \cos \theta \, d\omega \, dA \)

Radiance properties

- Radiance constant as propagates along ray
  - Derived from conservation of flux
  - Fundamental in Light Transport.

\[
\frac{d\Phi_1}{d\omega_1} = \frac{d\Phi_2}{d\omega_2} = \frac{d\Phi_1}{d\omega_1} \frac{dA_2}{dA_1} = \frac{d\omega_2}{d\omega_1} \frac{dA_2}{dA_1} \]

\[ \therefore L_1 = L_2 \]

Quiz

Does radiance increase under a magnifying glass?

No!!

Does the brightness that a wall appears to the eye depend on the distance of the viewer to the wall?

No!!
Radiance properties

- Sensor response proportional to radiance (constant of proportionality is throughput)
  - Far away surface: See more, but subtends smaller angle
  - Wall equally bright across viewing distances
Consequences
  - Radiance associated with rays in a ray tracer
  - Other radiometric quants derived from radiance

Irradiance, Radiosity

- Irradiance E is radiant power per unit area
- Integrate incoming radiance over hemisphere
  - Projected solid angle (cos \( \theta \) \( d\omega \))
  - Uniform illumination:
    - Irradiance = \( \pi \) [CW 24,25]
    - Units: W/m²
- Radiant Exitance (radiosity)
  - Power per unit area leaving surface (like irradiance)

Directional Power Arriving at a Surface

\[
d^2\Phi_i(x, \omega) = L_i(x, \omega) \cos \theta dA d\omega
\]

Irradiance from the Environment

\[
d^2\Phi_i(x, \omega) = L_i(x, \omega) \cos \theta dA d\omega
\]

Uniform Area Source

\[
E(x) = \int L \cos \theta d\omega
\]

Irradiance Environment Maps

Incident Radiance
(Illumination Environment Map)

Infrared Environment Map
Radiometry

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Types of Reflection Functions

- Ideal Specular
  - Reflection Law
  - Mirror
- Ideal Diffuse
  - Lambert’s Law
  - Matte
- Specular
  - Glossy
  - Directional diffuse

Materials

- Plastic
- Metal
- Matte

From Apodaca and Gritz, Advanced RenderMan

Spheres [Matusik et al.]

Building up the BRDF

- Bi-Directional Reflectance Distribution Function [Nicodemus 77]
- Function based on incident, view direction
- Relates incoming light energy to outgoing
- Unifying framework for many materials

The BRDF

Bidirectional Reflectance-Distribution Function

\[ f_i(\omega_i \rightarrow \omega_o) \approx \frac{dL_i(\omega_o) \cdot \frac{dE_o}{d\omega} \cdot \frac{d\omega_o}{d\omega}}{L_i(\omega_o)} \]
BRDF

- Reflected Radiance proportional Irradiance
- Constant proportionality: BRDF
- Ratio of outgoing light (radiance) to incoming light (irradiance)
  - Bidirectional Reflection Distribution Function
  - (4 Vars) units 1/sr
  \[
  f(\omega_o, \omega_i) = \frac{L(\omega_o)}{L(\omega_i) \cos \theta d\omega_i}
  \]
  \[
  L_i(\omega_i) = L_i(\omega_i) f(\omega_o, \omega_i) \cos \theta d\omega_i
  \]

Properties of BRDF’s

1. Linearity
2. Reciprocity principle

From Sillion, Arvo, Westin, Greenberg

Isotropic vs Anisotropic

- Isotropic: Most materials (you can rotate about normal without changing reflections)
- Anisotropic: brushed metal etc. preferred tangential direction

Energy Conservation

\[
\frac{d}{d\Omega} = \int_\Omega \frac{L_i(\omega_i) \cos \theta_i d\omega_i}{L(\omega_o) \cos \theta_o d\omega_o} = \int_\Omega f(\omega_o \rightarrow \omega_i) L_i(\omega_i) \cos \theta_i d\omega_i \cos \theta_o d\omega_o
\]

\[
\frac{d}{d\Omega} \leq 1
\]

Reflection Equation

\[
L_i(\omega_i) = L_i(\omega_i) + L_i(\omega_i) f(\omega_o, \omega_i)(\omega_i \cdot \omega_o)
\]
Reflection Equation

\[ L_r(x, \omega_r) = L_e(x, \omega_r) + \sum \int L_i(x, \varphi_i)(x, \varphi_i, \omega_i)(\varphi_i, \varphi_r) \cos \theta d\omega \]

Reflected Light
(Output Image)
Reflected Light
(Output Image)
Emission Incident Light (from light source)
BRDF Cosine of Incident angle
Sum over all light sources

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Brdf Viewer plots
- Diffuse
- Torrance-Sparrow
- Anisotropic

by written by Szymon Rusinkiewicz

Ideal Diffuse Reflection

Assume light is equally likely to be reflected in any output direction (independent of input direction).

\[ L_r(\omega_r) = \int f_r L_e(\omega) \cos \theta d\omega \]

\[ f_r L_e(\omega) \cos \theta d\omega = \int f_r L_e(\omega) \cos \theta d\omega = f_r L_e \]

\[ M = \int L_r(\omega) \cos \theta d\omega, \quad d\omega = L_e \cos \theta d\omega = \pi L_e \]

\[ \rho_r = \frac{M}{E} = \frac{\pi f_r L_e}{E} = \pi f_r \Rightarrow f_r = \frac{\rho_r E}{\pi} \]

Lambert’s Cosine Law
\[ M = \rho_r E, \quad \rho_r E \cos \theta \]

Phong Model

\[ \mathbf{E} \cdot \mathbf{R} = \mathbf{E} \cdot (\mathbf{R} \cdot \mathbf{E}) \]

Reciprocity: \[ \mathbf{E} \cdot \mathbf{R} = \mathbf{R} \cdot \mathbf{E} \]

Distributed light source
Specular Term (Phong)

Phong Model

Specular Term (Phong)

Fresnel Reflectance

Metal (Aluminum)

Dielectric (N=1.5)

Gold \( F(\theta) = 0.62 \)

Silver \( F(\theta) = 0.93 \)

Glasses \( n=1.5 \) \( F(\theta) = 0.04 \)

Diamond \( n=2.4 \) \( F(\theta) = 0.13 \)

Schlick Approximation

\[ F(\theta) = F(0) + (1 - F(0))(1 - \cos(\theta))^{5} \]

Torrance-Sparrow

Fresnel term: allows for wavelength dependency

Geometric Attenuation: reduces the output based on the amount of shadowing or masking that occurs.

\[ f = \frac{F(\theta_{i})G(\omega_{i}, \omega_{r})D(\theta_{r})}{4\cos(\theta_{i})\cos(\theta_{r})} \]

How much of the macroscopic surface is visible to the light source

How much of the macroscopic surface is visible to the viewer

Distribution: distribution function determines what percentage of microfacet are oriented to reflect in the viewer direction.

Experiment

Reflections from a shiny floor

From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

Other BRDF models

- Empirical: Measure and build a 4D table
- Anisotropic models for hair, brushed steel
- Cartoon shaders, funky BRDFs
- Capturing spatial variation
- Very active area of research