Computer Graphics II: Rendering

CSE 168 [Spr 21], Lecture 4: Rendering Equation
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http://viscomp.ucsd.edu/classes/cse168/sp21

To Do

- Homework 1 (ray tracer) due in a few days
- Next assignment direct lighting (on UCSD online). Will cover that material next week

Illumination Models

Local Illumination
- Light directly from light sources to surface
- No shadows (cast shadows are a global effect)

Global Illumination: multiple bounces (indirect light)
- Hard and soft shadows
- Reflections/refractions (already seen in ray tracing)
- Diffuse and glossy interreflections (radiosity, caustics)

Some images courtesy Henrik Wann Jensen

Caustics

Caustics: Focusing through specular surface

- Major research effort in 80s, 90s till today

Overview of lecture

- Theory for all global illumination methods (ray tracing, path tracing, radiosity)
- We derive Rendering Equation [Kajiya 86]
  - Major theoretical development in field
  - Unifying framework for all global illumination
  - Introduced Path Tracing: core rendering method
- Discuss existing approaches as special cases

Fairly theoretical lecture (but important). Not well covered in textbooks (though see Eric Veach’s thesis). See reading if you are interested.

Outline

- Reflectance Equation
- Global Illumination
- Rendering Equation
- As a general Integral Equation and Operator
- Approximations (Ray Tracing, Radiosity)
- Surface Parameterization (Standard Form)
**Reflection Equation**

\[ L_r(x, \omega_r) = L_e(x, \omega_i) + \int \Omega L_r(x', \omega_r') f(x, \omega_i, \omega_r') \cos \theta_i d\omega_i \]

- Reflected Light (Output Image)
- Emission
- BRDF
- Cosine of Incident angle

**Environment Maps**
- Light as a function of direction, from entire environment
- Captured by photographing a chrome steel or mirror sphere
- Accurate only for one point, but distant lighting same at other scene locations (typically use only one env. map)

**The Challenge**

- Computing reflectance equation requires knowing the incoming radiance from surfaces
- But determining incoming radiance requires knowing the reflected radiance from surfaces

**Rendering Equation**

\[ L_r(x, \omega_r) = L_e(x, \omega_i) + \int \Omega L_r(x', \omega_r') f(x, \omega_i, \omega_r') \cos \theta_i d\omega_i \]

- Reflected Light (Output Image)
- Emission
- BRDF
- Cosine of Incident angle

Blinn and Newell 1976, Miller and Hoffman, 1984
Later, Greene 86, Cabral et al. 87
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Rendering Equation (Kajiya 86)

Rendering Equation as Integral Equation

\[ L(x, \omega_i) = L(x, \omega_r) + \int \left[ L(x', \omega_r) f(x', \omega_i, \omega_r) \cos \theta \right] dv \]

Is a Fredholm Integral Equation of second kind [extensively studied numerically] with canonical form

\[ I(u) = e(u) + \int K(u, v) dv \]

Kernel of equation

Linear Operator Theory

- Linear operators act on functions like matrices act on vectors or discrete representations
  \[ h(u) = (M \circ f)(u) \]
  \( M \) is a linear operator,
  \( f \) and \( h \) are functions of \( u \)

- Basic linearity relations hold
  \( a \) and \( b \) are scalars
  \( f \) and \( g \) are functions

\[ M(af + bg) = a(Mf) + b(Mg) \]

Examples include integration and differentiation

\[ (K \circ f)(u) = \int K(u, v) f(v) dv \]

\[ (D \circ f)(u) = \frac{df}{du}(u) \]

Solving the Rendering Equation

- Too hard for analytic solution, numerical methods
- Approximations, that compute different terms, accuracies of the rendering equation
- Two basic approaches are ray tracing, radiosity. More formally, Monte Carlo and Finite Element. Today Monte Carlo path tracing is core rendering method

- Monte Carlo techniques sample light paths, form statistical estimate (example, path tracing)
- Finite Element methods discretize to matrix equation

\[ L = E + KL \]

Can be discretized to a simple matrix equation [or system of simultaneous linear equations]

\( L, E \) are vectors, \( K \) is the light transport matrix
Solving the Rendering Equation

- General linear operator solution: Within raytracing:
- General class numerical Monte Carlo methods
- Approximate set of all paths of light in scene

\[ L = E + KL \]
\[ IL - KL = E \]
\[ (I - K)\hat{L} = E \]

Binomial Theorem
\[ L = (I + K + K^2 + K^3 + \ldots)E \]
\[ L = E + KE + K^2E + K^3E + \ldots \]

Term \( n \) corresponds to \( n \) bounces of light

Ray Tracing

\[ L = E + KE + K^2E + K^3E + \ldots \]

Emission directly
From light sources
Direct Illumination
on surfaces
OpenGL Shading
Global Illumination
(One bounce indirect)
[Mirrors, Refraction]
(Two bounce indirect)
[Caustics etc]

Successive Approximation

- \( L_z \)
- \( \hat{L}_y = L_z \)
- \( L_z + K = \hat{L}_y \)
- \( L_z + K + L_z = \hat{L}_y \)
- \( L_z + \cdots K = \hat{L}_y \)

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Change of Variables

\[ L(x, \omega_i) = L_s(x, \omega_i) + \int L_r(x', \omega_i, \omega_i \omega_i) \cos \theta_i d \omega_i \]

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

\[ d \omega_i = \frac{dA' \cos \theta_i}{|x - x'|} \]

Change of Variables

\[ L(x, \omega_i) = L_s(x, \omega_i) + \int L_r(x', \omega_i, \omega_i \omega_i) \cos \theta_i d \omega_i \]

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

\[ d \omega_i = \frac{dA' \cos \theta_i}{|x - x'|} \]

\[ G(x, x') = G(x, x') = \frac{\cos \theta_i \cos \theta_i}{|x - x'|} \]

Rendering Equation: Standard Form

\[ L_s(x, \omega_i) = L_s(x, \omega_i) + \int L_r(x', \omega_i, \omega_i \omega_i) \cos \theta_i d \omega_i \]

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

\[ L_s(x, \omega_i) = L_s(x, \omega_i) + \int L_r(x', \omega_i, \omega_i \omega_i) \cos \theta_i d \omega_i \]

Domain integral awkward. Introduce binary visibility fn V

\[ L_s(x, \omega_i) = L_s(x, \omega_i) + \int_{\text{all visible}} L_r(x', \omega_i, \omega_i \omega_i) G(x, x') \pi \]

Same as equation 2.52 Cohen Wallace. It swaps primed and unprimed, omits angular args of BRDF, - sign.

Same as equation above 19.3 in Shirley, except he has no emission, slightly diff. notation

\[ G(x, x') = G(x, x') = \frac{\cos \theta_i \cos \theta_i}{|x - x'|} \]

Radiosity Equation

\[ L_s(x, \omega_i) = L_s(x, \omega_i) + \int_{\text{all surfaces}} L_r(x', \omega_i, \omega_i \omega_i) G(x, x') \pi \]

Drop angular dependence (diffuse Lambertian surfaces)

\[ L_s(x) = L_s(x) + \frac{1}{2} \int L_r(x', \omega_i) G(x, x') \pi \]

Change variables to radiosity (B) and albedo (\( \rho \))

\[ B(x) = E(x) + \rho(x) \int B(x') \frac{G(x, x') \pi}{\pi} \]

\[ B_i = E_i + \rho_i \sum_j B_j F_j \cdot \frac{A_j}{A} \]

F is the form factor. It is dimensionless and is the fraction of energy leaving the entirety of patch j (multiply by area of j to get total energy) that arrives anywhere in the entirety of patch i (divide by area of i to get energy per unit area or radiosity).
Matrix Equation

\[ B_i = E_i + \rho_i \sum_j B_j F_{j \rightarrow i} \frac{A_j}{A_i} \]

\[ A F_{i \rightarrow j} = A F_{j \rightarrow i} = \iint \frac{G(x, x') \mathcal{N}(x, x')}{\pi} dA_i dA_j \]

\[ B_i = E_i + \rho_i \sum_j B_j F_{j \rightarrow i} \]

\[ B_i - \rho_i \sum_j B_j F_{j \rightarrow i} = E_i \]

\[ \sum_j M_{ij} B_j = E_i \quad MB = E \quad M_{ij} = I_{ij} - \rho_i F_{i \rightarrow j} \]

Radiosity Epitaph

- Very hot topic (about 1985-1994). Some of the most beautiful images, greatest researchers; at one point 50% of SIGGRAPH papers
- But visibility, meshing, discontinuities, complex BRDFs, volumes were all difficult problems
- Since mid-90s, Monte Carlo (rather than finite element) methods were preferred (going back to Monte Carlo Path Tracing in Kajiya 86)
- Today, path tracing entirely method of choice, widely used in industry. This is what 168 teaches

Summary

- **Theory** for all global illumination methods (ray tracing, path tracing, radiosity)
- We derive **Rendering Equation** [Kajiya 86]
  - Major theoretical development in field
  - Unifying framework for all global illumination
- Discuss existing approaches as special cases
- **Rest of Course:** Solving the Rendering Equation (numerically using Monte Carlo Path Tracing)