Illumination Models

Local Illumination
- Light directly from light sources to surface
- No shadows (cast shadows are a global effect)

Global Illumination: multiple bounces (indirect light)
- Hard and soft shadows
- Reflections/refractions (already seen in ray tracing)
- Diffuse and glossy interreflections (radiosity, caustics)

Caustics

Caustics: Focusing through specular surface
- Major research effort in 80s, 90s till today

Overview of lecture

- **Theory** for all global illumination methods (ray tracing, path tracing, radiosity)
- We derive **Rendering Equation** [Kajiya 86]
  - Major theoretical development in field
  - Unifying framework for all global illumination
  - Introduced Path Tracing: core rendering method
- Discuss existing approaches as special cases

Outline

- **Reflectance Equation**
- **Global Illumination**
- **Rendering Equation**
- As a general Integral Equation and Operator
- Approximations (Ray Tracing, Radiosity)
- Surface Parameterization (Standard Form)
Reflection Equation

\[ L_r(x,\omega_i) = L_e(x,\omega_i) + \sum L_i(x,\omega_i) f(x,\omega_i,\omega_r)(\omega_r \cdot \omega) \]

Reflected Light (Output Image)
Emission
Incident Light (from light source)
BRDF
Cosine of Incident angle

Environment Maps

- Light as a function of direction, from entire environment
- Captured by photographing a chrome steel or mirror sphere
- Accurate only for one point, but distant lighting same at other scene locations (typically use only one env. map)

Blinn and Newell 1976, Miller and Hoffman, 1984
Later, Greene 86, Cabral et al. 87

The Challenge

- Computing reflectance equation requires knowing the incoming radiance from surfaces
- But determining incoming radiance requires knowing the reflected radiance from surfaces

Rendering Equation

\[ L_r(x,\omega_i) = L_r(x,\omega_i) + \int L_r(x',\omega_i) f(x,\omega_i,\omega_r) \cos \theta \, d\omega_i \]

Reflected Light (Output Image)
Emission
Reflected Light (from light source)
BRDF
Cosine of Incident angle

Surfaces (interreflection)
Outline

- Reflectance Equation (review)
- Global Illumination
- Rendering Equation
- As a general Integral Equation and Operator
- Approximations (Ray Tracing, Radiosity)
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Rendering Equation (Kajiya 86)

Rendering Equation as Integral Equation

\[ I(u) = e(u) + \int \int K(u, v) dv \]

Is a Fredholm Integral Equation of second kind [extensively studied numerically] with canonical form

Linear Operator Theory

- Linear operators act on functions like matrices act on vectors or discrete representations
- Basic linearity relations hold
  \[ a \text{ and } b \text{ are scalars; } f \text{ and } g \text{ are functions} \]
  \[ M \circ (af + bg) = a(M \circ f) + b(M \circ g) \]
- Examples include integration and differentiation
  \[ (K \circ f)(u) = \int K(u, v) f(v) dv \]
  \[ (D \circ f)(u) = \frac{df}{du}(u) \]

Linear Operator Equation

\[ I(u) = e(u) + \int \int K(u, v) dv \]

Kernel of equation

Light Transport Operator

\[ L = E + KL \]

Can be discretized to a simple matrix equation [or system of simultaneous linear equations]

(L, E are vectors, K is the light transport matrix)

Solving the Rendering Equation

- Too hard for analytic solution, numerical methods
- Approximations, that compute different terms, accuracies of the rendering equation
- Two basic approaches are ray tracing, radiosity. More formally, Monte Carlo and Finite Element. Today Monte Carlo path tracing is core rendering method

- Monte Carlo techniques sample light paths, form statistical estimate (example, path tracing)
- Finite Element methods discretize to matrix equation
Solving the Rendering Equation

- General linear operator solution: Within raytracing:
- General class numerical Monte Carlo methods
- Approximate set of all paths of light in scene

\[ L = E + KL \]
\[ IL - KL = E \]
\[ (I - K)L = E \]
\[ L = (I - K)^{-1}E \]

Binomial Theorem

\[ L = (I + K + K^2 + K^3 + ...)E \]
\[ L = E + KE + K^2E + K^3E + ... \]

Term \( n \) corresponds to \( n \) bounces of light

Ray Tracing

\[ L = E + KE + K^2E + K^3E + ... \]

Emission directly From light sources
Direct Illumination on surfaces
Global Illumination (One bounce indirect) [Mirrors, Refraction]
(Two bounce indirect) [Caustics etc]

OpenGL Shading

Successive Approximation

- \( L_0 \)
- \( K = L_0 \)
- \( K + K = L_0 \)
- \( K + K = L_0 \)

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- Reflectance Equation (review)
- Global Illumination
- Rendering Equation
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Rendering Equation

\[ L_0(x, \omega_i) = L_0(x, \omega_i) + \int \frac{dA}{d\omega_i} L_i(x', \omega_i - \omega_i) f(x, \omega_i, \omega_i) \cos \theta \ d\omega_i \]

Reflected Light (Output Image)
Emission
Reflected Light
BRDF
Cosine of Incident angle

UNKNOWN
KNOWN
UNKNOWN
KNOWN
KNOWN
Change of Variables

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

\[
\omega' = \cos \theta \frac{d\omega'}{|x - x'|}
\]

\[
\omega = \cos \theta \frac{d\omega}{|x - x'|}
\]

\[
G(x, x') = G(x', x) = \frac{\cos \theta \cos \theta}{|x - x'|}
\]

Rendering Equation: Standard Form

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

\[
L_i(x, \omega) = L_e(x, \omega) + \int_{s'} L_i'(x', -\omega') \eta(x', \omega') \cos \theta' d\omega'
\]

\[
L_i(x, \omega) = L_i(x, \omega) + \int_{s'} L_i'(x', \omega') G(x, x') \eta(x, \omega') \cos \theta' d\omega'
\]

\[
\omega' = \cos \theta \frac{d\omega'}{|x - x'|}
\]

\[
\omega = \cos \theta \frac{d\omega}{|x - x'|}
\]

Discretization and Form Factors

\[
B(x) = E(x) + \rho(x) \int_{s'} B(x') \frac{G(x, x') \eta(x, x')}{\pi} dA'
\]

\[
B_i = E_i + \rho_i \sum_j B_j F_{i-j} \frac{A_j}{A_i}
\]

F is the form factor. It is dimensionless and is the fraction of energy leaving the entirety of patch \( j \) (multiply by area of \( j \) to get total energy) that arrives anywhere in the entirety of patch \( i \) (divide by area of \( i \) to get energy per unit area or radiosity).

Radiosity Equation

Drop angular dependence (diffuse Lambertian surfaces)

\[
L_i(x) = L_e(x) + \int_{s'} L_i'(x', \omega') G(x, x') \eta(x, \omega') dA'
\]

Change variables to radiosity (\( B \)) and albedo (\( \rho \))

\[
B(x) = E(x) + \rho(x) \int_{s'} B(x') \frac{G(x, x') \eta(x, x')}{\pi} dA'
\]

Expresses conservation of light energy at all points in space

\[
\omega' = \cos \theta \frac{d\omega'}{|x - x'|}
\]

\[
\omega = \cos \theta \frac{d\omega}{|x - x'|}
\]

Form Factors

\[
A_i F_{i-j} = A_j F_{j-i} = \int G(x, x') \frac{\eta(x, x')}{\pi} dA_i dA_j
\]

\[
G(x, x') = G(x', x) = \frac{\cos \theta \cos \theta}{|x - x'|}
\]
Matrix Equation

\[ B_i = E_i + \rho_i \sum_j B_j F_{i\rightarrow j} \frac{A_j}{A_i} \]

\[ A_F_{i\rightarrow j} = A_{F_{i\rightarrow j}} = \int \frac{G(x,x')V(x',x)}{\pi} d\Omega dA_j \]

\[ B_i = E_i + \rho_i \sum_j B_j F_{i\rightarrow j} \]

\[ B_i - \rho_i \sum_j B_j F_{i\rightarrow j} = E_i \]

\[ \sum_j M_{ij} B_j = E_i \quad MB = E \quad M_{ij} = I_{ij} - \rho_j F_{i\rightarrow j} \]

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Radiosity Epitaph

- Very hot topic (about 1985-1994). Some of the most beautiful images, greatest researchers; at one point 50% of SIGGRAPH papers
- But visibility, meshing, discontinuities, complex BRDFs, volumes were all difficult problems
- Since mid-90s, Monte Carlo (rather than finite element) methods were preferred (going back to Monte Carlo Path Tracing in Kajiya86)
- Today, path tracing entirely method of choice, widely used in industry. This is what 168 teaches

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Summary

- **Theory** for all global illumination methods (ray tracing, path tracing, radiosity)
- We derive **Rendering Equation** [Kajiya 86]
  - Major theoretical development in field
  - Unifying framework for all global illumination
- Discuss existing approaches as special cases
- **Rest of Course**: Solving the Rendering Equation (numerically using Monte Carlo Path Tracing)