To Do

- Start working on final projects (initial results and proposal due in a week). Ask me if problems
- Volumetric rendering (this lecture) may be one component of the final project (but hard, be careful)
- Increasingly accurate appearance requires volumetric scattering (even for skin, hair, fur)
- Continues to be an active area of research

Many slides courtesy Pat Hanrahan/Matt Pharr (Stanford CS 348b) and Steve Rotenberg, Henrik Weir Jørgensen (UCSD CSE 168)

Volumetric Scattering

- Participating Media (light participates via scattering)
  - Volumetric phenomena like clouds, smoke, fire
  - Subsurface scattering, translucency (wax, human skin)
  - These are not surfaces with well-defined BRDFs
  - Rather volumes where light can scatter
  - Medium is often known as a participating medium
- Surface Rendering: Radiance Constant along Ray
  - Only true in absence of participating media
  - No longer true for volumetric scattering
  - Often replace ray tracing with ray marching in medium
- Volumetric Properties
  - BRDF replaced by phase function
  - Must consider absorption and scattering in medium

Homogeneous vs Heterogeneous

- Homogeneous: Properties constant everywhere
  - Example: Fog often represented as homogeneous
- Heterogeneous: Varies across space
  - Example: Smoke, fire etc.
  - Sometimes called inhomogeneous
- Homogeneous volumes often easier
  - Some computational shortcuts (transmittance etc.)
  - Some analytic formulae
Homogeneous vs Heterogeneous

Volumetric Interactions

- 4 different processes affect radiance of a beam
  - Absorption
  - Out-Scattering
  - Emission
  - In-Scattering

Absorption

\[ \frac{dL(p, \omega)}{L(p, \omega)} = -\sigma_a(p) L(p, \omega) \, ds \]

Absorption cross section: \( \sigma_a(p) \)

- Probability of being absorbed per unit length
- Units: \( 1 \text{/distance} \)

Transmittance

\[ \frac{dL(p, \omega)}{L(p, \omega)} = -\sigma_a(p) L(p, \omega) \, ds \]

\[ \log L(p + \kappa \omega, \omega) = -\int_0^s \sigma_a(p + s' \omega, \omega) \, ds' = -\tau(s) \]

Optical distance (depth): \( \tau(s) = \int_0^s \sigma_a(p') \, ds' \)

Homogeneous medium-constant \( \sigma_a \): \( \tau(s) = \sigma_a s \)

Transmittance and Opacity

\[ \frac{dL(p, \omega)}{L(p, \omega)} = -\sigma_a(p) L(p, \omega) \, ds \]

\[ \frac{dL(p, \omega)}{L(p, \omega)} = -\sigma_a(p) \, ds \]

\[ \log L(p + \kappa \omega, \omega) = -\int_0^s \sigma_a(p + s' \omega, \omega) \, ds' = -\tau(s) \]

\[ L(p + \kappa \omega, \omega) = e^{-\tau(s)} L(p, \omega) = T(s) L(p, \omega) \]

Transmittance: \( T(s) = e^{-\tau(s)} \)

Opacity: \( \alpha(s) = 1 - T(s) \)
Light interacts with volume, scatters in some spherical distribution
- Similar to light scattering off a surface
- Phase function analogous to a surface BRDF
- Depends only on cosine of incident-outgoing
- Like BRDFs, volumetric phase functions must be reciprocal and conserve energy
- Similar to BRDFs, we will want to do importance sampling and evaluation of phase functions
Phase Functions

- **Phase angle**: $\cos \theta = \omega \cdot \omega'$
- **Phase functions**
  - **Isotropic**: $p(\cos \theta) = \frac{1}{4\pi}$
  - **Rayleigh**: $p(\cos \theta) = \frac{3}{8} (1 + \cos^2 \theta) \text{ with } \phi \propto \frac{1}{\lambda^4}$
  - **Mie**:

Rayleigh Scattering

- Rayleigh scattering describes the scattering of light by particles much smaller than the wavelength
- The strong dependence on wavelength ($\lambda^4$) causes greater scattering towards the blue end of the spectrum
- The blue color of the sky is caused by Rayleigh scattering of sunlight by air molecules

\[
p(\cos \theta) = \frac{3}{16\pi} (1 + \cos^2 \theta)
\]

\[
\sigma_0 = \frac{2\pi^3 d^4}{3} \left( \frac{\lambda^2 - 1}{\pi^2 + 2} \right)^2
\]

Rayleigh Scattering: Blue Sky, Red Sunset

From Greenler: Rainbows, Halos, and Glories

Mie Scattering

- Scatter electromagnetic waves by spherical particles
- Size of particles same scale as wavelength of light
- Water droplets in atmosphere, fat droplets in milk
- After Gustave Mie, Ludvig Lorenz

Empirical Mie Approximation

- The following empirical function is often used to approximate the shape of Mie scattering

\[
p(\cos \theta) = \frac{1}{4\pi} \left( \frac{1}{2} + \frac{z + 1}{2} \left( 1 + \cos \theta \right)^{\frac{z}{2}} \right)
\]

Henyey-Greenstein Function

- The Henyey-Greenstein phase function is an empirical function originally designed to model the scattering in galactic dust clouds

\[
p(\cos \theta) = \frac{1 - g^2}{4\pi(1 + g^2 - 2g\cos \theta)^{1.5}}
\]

- It uses an anisotropy parameter $g$ that ranges between -1 (full backscatter) and 1 (full forward scatter), and is isotropic for $g=0$
Direct Illumination in a Volume

\[ S_i(p', \omega) = \sigma_i(p') \int_{S^2} p(\omega' \rightarrow \omega) L_i(p', \omega') \, d\omega' \]

- Can treat like direct illumination at a surface
- Sample from phase function’s distribution
- Sample from light source distributions
- Weight using multiple importance sampling

Transmittance for Shadow Rays

Besides Monte Carlo, precomputed transmittance can be faster for point, distant lights

Single-Scattering

The Volume Rendering Equation

Integro-differential equation:

\[ \frac{\partial L(p, \omega)}{\partial s} = -\sigma_i L(p, \omega) + S(p, \omega) \]

Integro-integral equation:

\[ L(p, \omega) = \int_0^\infty T(p') S(p', \omega) \, ds' \]

Attenuation: absorption and scattering

\[ e^{-\int_0^s \sigma_i(p') \, ds'} \]

Sources: in-scattering (and emission)

\[ \sigma_i(p') \int_{S^2} p(\omega' \rightarrow \omega) L(p', \omega') \, d\omega' \]
Volumetric Path Tracing

Integro-integral equation:
\[ L(p, \omega) = \int_0^\infty T(p') S(p', \omega) \, ds' \]
Monte Carlo integration: sample \( s' \sim p(s) \)
Estimator:
\[ \frac{T(p')S(p', \omega)}{p(s')} \]

Evaluating the Estimator: \( S \)

Include indirect illumination in the source term:
\[ S(x, \omega) = \sigma_s(x) \int_{S(x)} p(\omega' \to \omega) L(x, \omega') \, d\omega' \]
\[ L(x, \omega') = L_d(x, \omega') + L_i(x, \omega') \]
- Compute direct lighting as before
- Sample incident direction from the phase function's distribution, trace a ray recursively...
\[ L_i(x, \omega') \approx \frac{p(\omega'' \to \omega')} {p(\omega'')} L(x, \omega'') \]

Uniform spherical directions:
\[ p(\omega'') = \frac{1}{4\pi} \]

Linear Sampling of \( T \)

We want samples along a finite ray \([0, t_{max}]\).
- Uniform probability along the ray:
  \[ p(t) = \frac{1}{t_{max}} \]
- Sampling recipe:
  \[ \xi = \int_0^t p(t') \, dt' \]
  \[ t = \xi t_{max} \]

Exact Sampling of Uniform \( T \)

We want samples along a finite ray \([0, t_{max}]\), \( p(t) \propto e^{-\alpha t} \)
- Normalize to find PDF:
  \[ \int_0^{t_{max}} e^{-\alpha t} \, dt = \frac{1}{\alpha} (e^{-\alpha t_{max}} - 1) = c \]
  \[ p(t) = c e^{-\alpha t} \]
- Invert to find \( t \) for a random sample:
  \[ \xi = \int_0^t p(t') \, dt' \]
  \[ t = -\frac{1}{\alpha} \log(1 - \xi (1 - e^{-\alpha t_{max}})) \]

Volumetric Path Tracing

Integro-integral equation:
\[ L(p, \omega) = \int_0^\infty T(p') S(p', \omega) \, ds' \]
Monte Carlo integration: sample \( s' \sim p(s) \)
Estimator:
\[ \frac{T(p')S(p', \omega)}{p(s')} \]
Single-Scattering

Multiple Scattering

Clouds

Translucency
- Translucency is a volumetric lighting effect with additional effects at the surface (usually rough dielectric type interaction).
- These can be modeled through standard volumetric lighting techniques, or can be optimized through some further methods designed specifically for sub-surface scattering.

Fire
- “Physically Based Modeling and Animation of Fire”, Nguyen, Fedkiw, Jensen, 2003

Sky Rendering
- “A Practical Analytical Model for Daylight”, Preetham, Shirley, Smits, 1999
- “Precomputed Atmospheric Scattering”, Bruneton, Neyret, 2008
Volumetric Caustics


Rainbows

- “Physically Based Simulation of Rainbows”, Sadeghi, Munoz, Laven, Jarosz, Seron, Gutierrez, Jensen, 2012

Atmospheric Phenomena

- Corona
- Ice Crystal Halo
- Glory