To Do

- Homework 4 (importance sampling) due May 18
- These lectures cover more advanced topics
  - May be relevant for your final project
  - Or curiosity in terms of frontiers of modern rendering

Motivation

- Rendering Equation since 86, Path Tracer in HW 3
- So, is Monte Carlo rendering solved?
- Can it be made more efficient (90s until today)?
  - Multiple Importance Sampling (Homework 4)
  - Irradiance Caching takes advantage of coherence
  - Correct sampling: Stratified, Multiple Importance, Bidirectional Path Tracing, Metropolis, VCM/UPS, …
- Photon Mapping
- Modern adaptive sampling, cut-based integration
- Advanced topics (next time)
- Denoising (next time)

Smoothness of Indirect Lighting

- Direct
- Indirect
- Direct + Indirect

Irradiance Caching

- Empirically, (diffuse) interreflections low frequency
- Therefore, should be able to sample sparsely
- Irradiance caching samples irradiance at few points on surfaces, and then interpolates
- Ward, Rubinstein, Clear. SIGGRAPH 88, A ray tracing solution for diffuse interreflection

Irradiance Calculation

\[ E(x) = \sum_{i} w(x_i) E_i(x_i) \]

\[ E(x) = \int L_i(x, \omega) \cos \theta \, d\omega \]

\[ E(x) \leq \frac{1}{\pi} \left( \frac{1}{\sqrt{2 - 2 \cdot \hat{N}(x) \cdot \hat{N}(x_0)}} \right) \]

\[ \frac{\partial E}{\partial x} (x - x_0) + \frac{\partial E}{\partial \theta} (\theta - \theta_0) \]

Derivation in Ward paper
Algorithm Outline

- Find all samples with \( w(x) > q \)
- if (samples found)
  - interpolate
- else
  - compute new irradiance
- N.B. Subsample the image first and then fill in

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Better Sampling

- Smarter ways to Monte Carlo sample
- Long history: Stratified, Importance, Bi-Directional, Multiple Importance, Metropolis
- Good reference is Veach thesis
- We only briefly discuss a couple of strategies

Stratified Sampling

Stratified sampling like jittered sampling
Allocate samples per region
\[
N = \sum_{r} N_r, \quad F_r = \frac{1}{N} \sum_{i} N_i F_i
\]
New variance
\[
\mathbb{V}[F_r] = \frac{1}{N} \sum_{i} \mathbb{V}[F_i]
\]
Thus, if the variance in regions is less than the overall variance, there will be a reduction in resulting variance
For example: An edge through a pixel
\[
\mathbb{V}[F_r] = \frac{1}{N} \sum_{i} \mathbb{V}[F_i] = \frac{\mathbb{V}[F]}{1/N}
\]

Comparison of simple patterns

16 samples for area light, 4 samples per pixel, total 64 samples
If interested, see my paper “A Theory of Monte Carlo Visibility Sampling”

Figures courtesy: Tianyu Liu
**Spectrally Optimal Sampling**

Mitchell 91

**Light Ray Tracing**

Backwards Ray Tracing

[Arvo 86]

**Path Tracing: From Lights**

- Step 1. Choose a light ray
- Step 2. Find ray-surface intersection
- Step 3. Reflect or transmit
  
  \[ u = \text{Uniform()} \]
  
  if \( u < \text{reflectance}(x) \)
  
  Choose new direction \( d \sim \text{BRDF}(O|I) \)
  
  goto Step 2
  
  else if \( u < \text{reflectance}(x) + \text{transmittance}(x) \)
  
  Choose new direction \( d \sim \text{BTDF}(O|I) \)
  
  goto Step 2
  
  else // absorption = 1 – reflectance – transmittance
  
  terminate on surface; deposit energy

**Bidirectional Path Tracing**

Path pyramid (\( k = l + e = \text{total number of bounces} \))

\[ k = 3 \]

\[ k = 4 \]

\[ k = 5 \]

\[ k = 6 \]

\( (l=2, e=1) \)

\( (l=5, e=1) \)

[From Veach and Guibas]

**Space-time Patterns**

<table>
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<tr>
<th>x</th>
<th>y</th>
<th>z</th>
<th>w</th>
</tr>
</thead>
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<td>2</td>
<td>14</td>
</tr>
<tr>
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<td>4</td>
<td>8</td>
<td>12</td>
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<td>15</td>
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<td>7</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

**Cook Pattern**

- Fully populate \( (x,y) \) samples
- Recall blue noise good
- Perceptually pleasing
- Filtered during resampling
- Jitter to achieve blue noise

**Distribute / samples**

- Decorrelate space and time
- Nearby samples in space should differ greatly in time

**Mitchell (1991) designs**

**Comparison**

Bidirectional path tracing

Path tracing

[From Veach and Guibas]
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Why Photon Map?
- Some visual effects like caustics hard with standard path
  tracing from eye
- May usually miss light source altogether
- Instead, store “photons” from light in kd-tree
- Look-up into this as needed
- Combines tracing from light source, and eye
- Similar to bidirectional path tracing, but compute photon
  map only once for all eye rays
- Global illumination using Photon Maps H. Jensen,
  Rendering Techniques (EGSR 1996), pp 21–30. (Also

Caustics
Path Tracing: 1000 paths/pixel
Note noise in caustics

Reflections Inside a Metal Ring
50000 photons
50 photons to estimate radiance

Caustics on Glossy Surfaces
340000 photons, 100 photons in radiance estimate

Caustics
Photon Mapping: 10000 photons
50 photons in radiance estimate
HDR Environment Illumination

Global Illumination

Direct Illumination

Specular Reflection

Caustics

Indirect Illumination
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Complex Lighting

- Simulate complex illumination using point lights
  - Area lights
  - HDR environment maps
  - Sun & sky light
  - Indirect illumination
- Unifies illumination
  - Enables tradeoffs between components

Lightcuts

- Efficient, accurate complex illumination
  - Environment map lighting & indirect
    Time 111s
  - Textured area lights & indirect
    Time 98s
(640x480, Anti-aliased, Glossy materials)
From Walter et al. SIGGRAPH 05

Cornell Box

- 200,000 global photons, 50,000 caustic photons

Box: Global Photons

Mies House: Swimming Pool

- Environment map lighting & indirect
- Textured area lights & indirect
- Sun & sky light
- Indirect illumination

Complex Lighting

- Unifies illumination
  - Enables tradeoffs between components

Area lights + Sun/sky + Indirect

Environment map lighting & indirect

Textured area lights & indirect

(640x480, Anti-aliased, Glossy materials)

From Walter et al. SIGGRAPH 05
**Key Concepts**

- Light Cluster
- Light Tree
  - Binary tree of lights and clusters

**Clusters**

**Individual Lights**

**Simple Example**

1. Light Tree
2. Representative Light
3. Clusters
4. Individual Lights

**Three Example Cuts**

- A Cut
  - A set of nodes that partitions the lights into clusters

**Three Example Cuts**

- #1 #2 #4
- #1 #3 #4
- #1 #4

**Three Cuts**

- Good
- Bad
- Bad

**Three Cuts**

- Bad
- Good
- Bad
Three Example Cuts

Tableau, 630K polygons, 13,000 lights, (EnvMap+Indirect)
Avg. shadow rays per eye ray 17 (0.13%)