Outline

• What is polar form?
• B-Spline
  – Labeling, why (0,1,2), (1,2,3)… ?
  – Knot Vector, why (-2,-1,0,1,2,...) ?
• Derivation of B-Spline points
  – Maybe(Next Week?)
What is Polar Form Labeling?

- The sequences of parameters applied at each level of pyramid for de Casteljau evaluation.

\[ P_{\text{new}} = (u_0, u_1) \]
Polar Form for Quadratic Bezier

\[ P_{\text{new}} = (u_0, u_1) \]
Quiz

\((0, 0, 0) = P_0\)
\((0, 1, 1) = P_2\)
\((u, 0, 1) = B\)

\((0, 0, 1) = ?\)
\((u, 1, 1) = ?\)
\((u, u, 0) = ?\)
\((u, 0, 0) = ?\)
\((u, u, u) = ?\)
(0, 0, 0) = P0
(0, 1, 1) = P2
(0, 0, 0) = ?
(0, 0, 1) = ?
(u, 0, 1) = ?
(u, u, 0) = ?
(u, u, 1) = ?
(u, u, u) = ?
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de Casteljau weights (how?)
in B-Spline

\[
\begin{array}{ccccccccc}
-2 & -1 & 0 & -1 & 0 & 1 & 0 & 1 & 2 & 1 & 2 & 3 \\
\end{array}
\]

\[
\begin{array}{ccccccccc}
-1 & 0 & u & 0 & 1 & u & 1 & 2 & u \\
\end{array}
\]

\[
\begin{array}{ccccccccc}
1-u/3 & (1-u)/3 & (2+u)/3 & (2-u)/3 & (1+u)/3 & u/3 \\
\end{array}
\]

\[
\begin{array}{ccccccccc}
(1-u)/2 & (1+u)/2 & 1-u/2 & u/2 & \\
\end{array}
\]

\[
\begin{array}{ccccccccc}
0 & u & 0 & u & 1 & u & 1 & u & \\
\end{array}
\]

\[
\begin{array}{ccccccccc}
1-u & u & 1-u & u & \\
\end{array}
\]

\[
\begin{array}{ccccccccc}
u & u & u & u & \\
\end{array}
\]
Interpolating Weights for Bspline

\[(x, A, B) \quad u \quad (y, A, B)\]

\[(x, A, B) \quad (y, A, B) \quad ? \quad ? \quad (u, A, B)\]
Interpolating Weights for Bspline

\[(x, A, B) \quad u \quad (y, A, B)\]

\[
\begin{align*}
(u - x) & \\
(y - u) & \\
(y - x) &
\end{align*}
\]
Example

\[(u-x) \quad (y-u) \quad (y-x)\]
Example

\[ (1, 0, 1) \rightarrow u \rightarrow (0, 1, 2) \]

\[ (u - (-1)) = u + 1 \]

\[ (2 - u) \]

\[ (2 - (-1)) = 2 + 1 = 3 \]

\[ (-1, 0, 1) \rightarrow ? \rightarrow \frac{(2-u)}{3} \]

\[ (u + 1) \rightarrow ? \rightarrow \frac{(u + 1)}{3} \]
Weights for quadratic B-Spline
Example

\[(u - (-1)) = u + 1\]

\[(1 - u)\]

\[(1 - (-1)) = 1 + 1 = 2\]

\((1 - u)\) \(\frac{(u + 1)}{2}\)

\((-1, 0)\) \((0, 1)\)
Weights for quadratic B-Spline

\[
\begin{array}{ccc}
-1 & 0 & 1 \\
(1-u)/2 & (u+1)/2 & (2-u)/2 \\
0 & u & 1 \\
(1-u) & u/2 & u \\
0 & u & 1 \\
u & u \\
\end{array}
\]
So why B-Spline and how?

• Smoothness by joining Bezier curves
  – 3 points must be in the same line
  – Exact spacing
  – Hard to control

• Labeling
  – How to get the non-symmetric labeling
  – Knot-vector
Connecting Bezier Curves

$u_0 = 0$
$u_1 = 1$

$u_0 = 1,$
$u_1 = 0$
Remember Bezier Subdivision

\[ u = 0 \]

\[ u = 0.5 \]

\[ u = 1 \]
Uniform re-parameterization
But Hard to Control !!!
Need to control all 3 simultaneously
Controllability

$u = 0$ $u = 1$ $u = 2$
And...you can skip the derivation if you want
Adding the 3\textsuperscript{rd} Segment
Reverse Subdivision
Adding New Control

\[ u = 2 \quad u = 0 \quad u = 1 \quad u = 2 \quad u = 3 \]
Prepend Segments $u=-1\sim0$
Add the New Control
Prepend new Segment $u = -2\sim-1$
Add new control

Should be mid point, sorry for bad drawing
OK, stop, wake up
Here’s the “BOX,” Control Points

should be mid point, sorry for bad drawing
If you want only $u = 0 \sim 1$
Labeling from 2 Bezier curves
Now you can skip derivation for all labeling…
OK, we’re done.
Here’s the final labeling
Extensions

- B-Spline to Bezier
- Bezier to B-Spline
If you stare at the graph for a long time…

Should be mid point, sorry for bad drawing
Easy conversion from B-Spline and Bezier
Bezier to Bspline
Bezier to Bspline
Bezier to Bspline
Easy conversion from B-Spline and Bezier
Bezier to B-Spline
Bezier to B-Spline