Advanced Computer Graphics
CSE 163 [Spring 2018], Lecture 11
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To Do
- Assignment 2 due May 18
  - Should already be well on way.
  - Contact us for difficulties etc.
- This lecture on rendering, rendering equation. Pretty advanced theoretical material. Don’t worry if a bit lost; not directly required on the homeworks.

Course Outline
- 3D Graphics Pipeline
  - Rendering (Creating, shading images from geometry, lighting, materials)
- Modeling (Creating 3D Geometry)

Unit 1: Foundations of Signal and Image Processing
Understanding the way 2D images are formed and displayed, the important concepts and algorithms, and to build an image processing utility like Photoshop
Weeks 1 – 3. Assignment 1

Unit 2: Meshes, Modeling
Weeks 3 – 5. Assignment 2

Unit 3: Advanced Rendering
Weeks 6 – 7.8-9. (Final Project)

Unit 4: Animation, Imaging
Weeks 7-8, 9-10. (Final Project)

Illumination Models
- Local Illumination
  - Light directly from light sources to surface
  - No shadows (cast shadows are a global effect)
- Global Illumination: multiple bounces (indirect light)
  - Hard and soft shadows
  - Reflections/refractions (already seen in ray tracing)
  - Diffuse and glossy interreflections (radiosity, caustics)

Diffuse Interreflection
- Diffuse interreflection, color bleeding [Cornell Box]

Some images courtesy Henrik Wann Jensen
Overview of lecture

- **Theory** for all global illumination methods (ray tracing, path tracing, radiosity)
- We derive **Rendering Equation** [Kajiya 86]
- Major theoretical development in field
- Unifying framework for all global illumination
- Discuss existing approaches as special cases

Outline

- **Reflectance Equation**
- **Global Illumination**
- **Rendering Equation**
  - As a general Integral Equation and Operator
  - Approximations (Ray Tracing, Radiosity)
  - Surface Parameterization (Standard Form)

Reflection Equation

\[
L_r(x, \omega_r) = L_e(x, \omega_r) + \sum L_i(x, \omega_i) f(x, \omega_i, \omega_r)(\omega_i \cdot n)
\]

- Reflected Light (Output Image)
- Emission
- Incident Light (from light source)
- BRDF
- Cosine of Incident angle

Sum over all light sources
Reflection Equation

\[ L_r(x, \omega_r) = L_e(x, \omega_i) + \int_{\Omega} L_e(x, \omega_i)f(x, \omega_i, \omega_r) \cos \theta \, d\omega_i \]

Environment Maps

- Light as a function of direction, from entire environment
- Captured by photographing a chrome steel or mirror sphere
- Accurate only for one point, but distant lighting same at other scene locations (typically use only one env. map)

Environment Maps

- Environment maps widely used as lighting representation
- Many modern methods deal with offline and real-time rendering with environment maps
- Image-based complex lighting + complex BRDFs

The Challenge

\[ L_r(x, \omega_r) = L_e(x, \omega_i) + \int_{\Omega} L_e(x, \omega_i)f(x, \omega_i, \omega_r) \cos \theta \, d\omega_i \]

- Computing reflectance equation requires knowing the incoming radiance from surfaces
- But determining incoming radiance requires knowing the reflected radiance from surfaces

Outline

- Reflectance Equation (review)
- Global Illumination
- Rendering Equation
- As a general Integral Equation and Operator
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Rendering Equation (Kajiya 86)

Figure 6. A sample image. All objects are 3D. Order in the objects depends on the objects prior to casting. The picture goes blue and color bleeding from the scene is shown.

Rendering Equation as Integral Equation

\[ L(x, \omega_r) = E(x, \omega_r) + \int \frac{d}{d \omega_i} \left( \frac{f(x, \omega_i, \omega_r) \cos \theta_{\omega_i}}{d \omega_i} \right) \]

Is a Fredholm Integral Equation of second kind [extensively studied numerically] with canonical form

\[ l(u) = e(u) + \int I(v) K(u, v) dv \]

Kernel of equation

Linear Operator Theory

- Linear operators act on functions like matrices act on vectors or discrete representations

\[ h(u) = (M \circ f)(u) \quad M \text{ is a linear operator, } f \text{ and } h \text{ are functions of } u \]

- Basic linearity relations hold

\[ a \cdot f + b \cdot g = a(M \circ f) + b(M \circ g) \]

- Examples include integration and differentiation

\[ (K \circ f)(u) = \int K(u, v) f(v) dv \]

\[ (D \circ f)(u) = \frac{d}{du} f(u) \]

Linear Operator Equation

\[ l(u) = e(u) + \int I(v) K(u, v) dv \]

Kernel of equation

Light Transport Operator

\[ L = E + KL \]

Can be discretized to a simple matrix equation [or system of simultaneous linear equations]

\( (L, E \text{ are vectors, } K \text{ is the light transport matrix}) \)

Solving the Rendering Equation

- Too hard for analytic solution, numerical methods
- Approximations, that compute different terms, accuracies of the rendering equation
- Two basic approaches are ray tracing, radiosity. More formally, Monte Carlo and Finite Element
- Monte Carlo techniques sample light paths, form statistical estimate (example, path tracing)
- Finite Element methods discretize to matrix equation

Solving the Rendering Equation

- General linear operator solution. Within raytracing:
- General class numerical Monte Carlo methods
- Approximate set of all paths of light in scene

\[ L = E + KL \]

\[ (L - KL) = E \]

\[ E = (I - K)^{-1} E \]

Binomial Theorem

\[ L = (1 + K + K^2 + K^3 + \ldots) E \]

\[ L = E + KE + K^2E + K^3E + \ldots \]

Term \( n \) corresponds to \( n \) bounces of light
Ray Tracing

\[ L = E + KE + K^2E + K^3E + \ldots \]

- Emission directly from light sources
- Direct illumination on surfaces
- Global illumination (One bounce indirect) [Mirs, Refraction]
- [Two bounce indirect] [Caustics etc]

OpenGL Shading

- Emission directly from light sources
- Direct illumination on surfaces
- Global illumination (One bounce indirect) [Mirs, Refraction]
- [Two bounce indirect] [Caustics etc]

Successive Approximation

- \( L_0 \)
- \( K \cdot L_0 \)
- \( K + K \cdot L_0 \)
- \( K + K \cdot K + L_0 \)
- \( L_0 + \ldots K^2 + L_0 \)
- \( L_0 + \ldots K^3 + L_0 \)

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Rendering Equation

\[ L_r(x, \omega_r) = E_r(x, \omega_r) + \int \int L_r(x', \omega_r') f(x, \omega_r, \omega_r') \cos \theta d\omega_r \]

Integral over angles sometimes insufficient. Write integral in terms of surface radiances only (change of variables)

\[ d\omega_r = \frac{dA' \cos \theta_r}{|x' - x|^2} \]
**Change of Variables**

Integral over angles sometimes insufficient. Write integral in terms of surface radiances only (change of variables).

\[ L_i(x, o_i) = L_o(x, o_i) + \int_{\Omega'} L_i(x', -o_i) f(x, o_i, o_i) \cos \theta d\omega \]

**Rendering Equation: Standard Form**

Integral over angles sometimes insufficient. Write integral in terms of surface radiances only (change of variables).

\[ L_i(x, o_i) = L_o(x, o_i) + \int_{\Omega'} L_i(x', -o_i) f(x, o_i, o_i) \cos \theta d\omega \]

**Radiosity Equation**

Integral over angles sometimes insufficient. Write integral in terms of surface radiances only (change of variables).

\[ L_i(x, o_i) = L_o(x, o_i) + \int_{\Omega'} L_i(x', -o_i) f(x, o_i, o_i) G(x, x') dA' \]

Drop angular dependence (diffuse Lambertian surfaces)

\[ L_i(x) = L_o(x) + f(x) \int_{\Omega'} G(x, x') dA' \]

Change variables to radiosity (B) and albedo (ρ)

\[ B(x) = E(x) + \rho(x) \int_{\Omega'} G(x, x') dA' \]

Expresses conservation of light energy at all points in space

Same as equation 2.54 in Cohen Wallace handout (read sec 2.6.3) ignore factors of π which can be absorbed.

**Discretization and Form Factors**

\[ B_i = E_i + \rho_i \sum_j B_j F_{i-j} \frac{A_j}{A_i} \]

F is the form factor. It is dimensionless and is the fraction of energy leaving the entirety of patch j \((\text{multiply by area of j})\) to get total energy that arrives anywhere in the entirety of patch i \((\text{divide by area of i to get energy per unit area or radiosity}).\)

\[ A_i F_{i-j} = A_j F_{j-i} = \int G(x, x') dA_i dA_j \]

\[ G(x, x') = G(x', x) = \frac{\cos \theta \cos \theta'}{|x - x'|^2} \]

\[ \sum_j M_j B_j = E_i \quad MB = E \quad M_i = I_i - \rho_i F_{i-j} \]

**Form Factors**

\[ A_i F_{i-j} = A_j F_{j-i} = \int G(x, x') dA_i dA_j \]

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