Motivation
- A polygon mesh is a collection of triangles
- We want to do operations on these triangles
  - E.g. walk across the mesh for simplification
  - Display for rendering
  - Computational geometry
- Best representations (mesh data structures)?
  - Compactness
  - Generality
  - Simplicity for computations
  - Efficiency

Mesh Data Structures
Desirable Characteristics 1
- Generality – from most general to least
  - Polygon soup
  - Only triangles
  - 2-manifold: ≤ 2 triangles per edge
  - Orientable: consistent CW/CCW winding
  - Closed: no boundary
- Compact storage

Outline
- Independent faces
- Indexed face set
- Adjacency lists
- Winged-edge
- Half-edge

Overview of mesh decimation and simplification
Independent Faces

Faces list vertex coordinates
- Redundant vertices
- No topology information

Indexed Face Set

- Faces list vertex references – "shared vertices"
- Commonly used (e.g. OFF file format itself)
- Augmented versions simple for mesh processing

Indexed Face Set

- Storage efficiency?
- Which operations supported in \( O(1) \) time?

Efficient Algorithm Design

- Can sometimes design algorithms to compensate for operations not supported by data structures
  - Example: per-vertex normals
    - Average normal of faces touching each vertex
    - With indexed face set, vertex \( \rightarrow \) face is \( O(n) \)
    - Naive algorithm for all vertices: \( O(n^2) \)
    - Can you think of an \( O(n) \) algorithm?

Efficient Algorithm Design

- Useful to augment with vertex \( \rightarrow \) face adjacency
  - For all vertices, find adjacent faces as well
  - Can be implemented while simply looping over faces

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Overview of mesh decimation and simplification
**Full Adjacency Lists**

- Store all vertex, face, and edge adjacencies

**Edge Adjacency Table**

- $e_0$: $v_0, v_1, F_0, ø, ø, e_2, e_1, ø$
- $e_1$: $v_1, v_2, F_0, F_1, e_5, e_0, e_2, e_6$

**Face Adjacency Table**

- $F_0$: $v_0, v_1, v_2, F_1, ø, ø, e_0, e_2, e_1$
- $F_1$: $v_1, v_4, v_2, ø, F_0, F_2, e_6, e_1, e_5$
- $F_2$: $v_1, v_3, v_4, ø, F_1, ø, e_4, e_5, e_3$

**Vertex Adjacency Table**

- $v_0$: $v_1, v_2, F_0, ø, e_0, e_2$
- $v_1$: $v_3, v_4, v_2, v_0, F_2, F_1, F_0, e_3, e_5, e_1, e_0$

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**Full adjacency: Issues**

- Garland and Heckbert claim they do this
- Easy to find stuff
- Issue is storage
- And updating everything once you do something like an edge collapse for mesh simplification
- I recommend you implement something simpler (like indexed face set plus vertex to face adjacency)

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**Partial Adjacency Lists**

- Store some adjacencies, use to derive others
- Many possibilities...

**Edge Adjacency Table**

- $e_0$: $v_0, v_1, F_0, ø, ø, e_2, e_1, ø$

**Face Adjacency Table**

- $F_0$: $v_0, v_1, v_2, F_1, ø, ø, e_0, e_2, e_1$
- $F_1$: $v_1, v_4, v_2, ø, F_0, F_2, e_6, e_1, e_5$
- $F_2$: $v_1, v_3, v_4, ø, F_1, ø, e_4, e_5, e_3$

**Vertex Adjacency Table**

- $v_0$: $v_1, v_2, F_0, ø, e_0, e_2$
- $v_1$: $v_3, v_4, v_2, v_0, F_2, F_1, F_0, e_3, e_5, e_1, e_0$

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**Partial Adjacency Lists**

- Some combinations only make sense for closed manifolds

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Overview of mesh decimation and simplification

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**Winged, Half Edge Representations**

- Idea is to associate information with edges
- Compact Storage
- Many operations efficient
- Allow one to walk around mesh
- Usually general for arbitrary polygons (not triangles)
- But implementations can be complex with special cases relative to simple indexed face set++ or partial adjacency table
Winged Edge

- Most data stored at edges
- Vertices, faces point to one edge each

Edge Adjacency Table

$e_0$: $v_0, v_1, F_0, F_2, e_2, e_1, ø
$e_1$: $v_1, v_2, F_0, F_1, e_5, e_0, e_2, e_6

Face Adjacency Table

$F_0$: $v_0, v_1, v_2, F_1, ø, ø, e_0, e_2, e_1
$F_1$: $v_1, v_4, v_2, ø, F_0, F_2, e_6, e_1, e_5
$F_2$: $v_1, v_3, v_4, ø, F_1, ø, e_4, e_5, e_3

Vertex Adjacency Table

$v_0$: $v_1, v_2, F_0, e_0, e_2
$v_1$: $v_3, v_4, v_2, v_0, F_2, F_1, F_0, e_3, e_5, e_1, e_0

Half Edge

- Instead of single edge, 2 directed “half edges”
- Makes some operations more efficient
- Walk around face very easily (each face need only store one pointer)

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Overview of mesh decimation and simplification

Mesh Decimation

- Reduce number of polygons
  - Less storage
  - Faster rendering
  - Simpler manipulation
- Desirable properties
  - Generality
  - Efficiency
  - Produces “good” approximation

Mesh Decimation

- Triangles: 41,855
  - 27,970
  - 20,922
  - 12,939
  - 8,935
  - 4,766

Michelangelo’s St. Matthew
- Original model: ~400M polygons
**Primitive Operations**

Simplify model a bit at a time by removing a few faces
- Repeated to simplify whole mesh

Types of operations
- Vertex cluster
- Vertex remove
- Edge collapse (main operation used in assignment)

**Vertex Cluster**

- Method
  - Merge vertices based on proximity
  - Triangles with repeated vertices can collapse to edges or points
- Properties
  - General and robust
  - Can be unattractive if results in topology change

**Vertex Remove**

- Method
  - Remove vertex and adjacent faces
  - Fill hole with new triangles (reduction of 2)
- Properties
  - Requires manifold surface, preserves topology
  - Typically more attractive
  - Filling hole well not always easy

**Edge Collapse**

- Method
  - Merge two edge vertices to one
  - Delete degenerate triangles
- Properties
  - Special case of vertex cluster
  - Allows smooth transition
  - Can change topology

**Mesh Decimation/Simplification**

Typical: greedy algorithm
- Measure error of possible “simple” operations (primarily edge collapses)
- Place operations in queue according to error
- Perform operations in queue successively (depending on how much you want to simplify model)
- After each operation, re-evaluate error metrics

**Geometric Error Metrics**

- Motivation
  - Promote accurate 3D shape preservation
  - Preserve screen-space silhouettes and pixel coverage
- Types
  - Vertex-Vertex Distance
  - Vertex-Plane Distance
  - Point-Surface Distance
  - Surface-Surface Distance
### Vertex-Vertex Distance

- $E = \max(v3-v1, |v3-v2|)$
- Appropriate during topology changes
  - Rossignac and Borrel 93
  - Luebke and Erikson 97
- Loose for topology-preserving collapses

![Vertex-Vertex Distance Diagram](image)

### Vertex-Plane Distance

- Store set of planes with each vertex
  - Error based on distance from vertex to planes
  - When vertices are merged, merge sets
- Ronfard and Rossignac 96
  - Store plane sets, compute max distance
- Error Quadrics – Garland and Heckbert 97
  - Store quadric form, compute sum of squared distances

![Vertex-Plane Distance Diagram](image)

### Point-Surface Distance

- For each original vertex, find closest point on simplified surface
- Compute sum of squared distances

![Point-Surface Distance Diagram](image)

### Surface-Surface Distance

- Compute or approximate maximum distance between input and simplified surfaces
  - Tolerance Volumes - Guéziec 96
  - Simplification Envelopes - Cohen/Varshney 96
  - Hausdorff Distance - Klein 96
  - Mapping Distance - Bajaj/Schikore 96, Cohen et al. 97

![Surface-Surface Distance Diagram](image)

### Geometric Error Observations

- Vertex-vertex and vertex-plane distance
  - Fast
  - Low error in practice, but not guaranteed by metric
- Surface-surface distance
  - Required for guaranteed error bounds

![Geometric Error Observations Diagram](image)

### Mesh Simplification

**Advanced Considerations**

- Type of input mesh?
- Modifies topology?
- Continuous LOD?
- Speed vs. quality?
**View-Dependent Simplification**

- Simplify dynamically according to viewpoint
  - Visibility
  - Silhouettes
  - Lighting

**Appearance Preserving**

- Compact storage vs ease, efficiency of use
- How fast and easy are key operations
- Mesh simplification
  - Reduce size of mesh in efficient quality-preserving way
  - Based on edge collapses mainly
- Choose appropriate mesh data structure
  - Efficient to update, edge-collapses are local

**Summary**

- Many mesh data structures
  - Compact storage vs ease, efficiency of use
  - How fast and easy are key operations
- Mesh simplification
  - Reduce size of mesh in efficient quality-preserving way
  - Based on edge collapses mainly
- Choose appropriate mesh data structure
  - Efficient to update, edge-collapses are local
- Fairly modern ideas (last 15-20 years)
  - Think about some of it yourself, see papers given out
  - We will cover simplification, quadric metrics next