

15/9/2015 Russell's lec 6. Graph algorithms: Faster APSP via circ comp.
 APSP: given n -node G , defined by weight w . $w: n \times n \rightarrow \mathbb{R} \cup \{\infty\}$.

Constraint: matrix S , s.t. $S[i,j] = \min_k \{w[i,k] + S[k,j]\}$. Fastest path from i to j starting w/ edge (i,k) .

\rightarrow success exists and can be described in $O(n^2 \log n)$ bits.

Value: weights are 2 $\log_2 n$ bits each. Other: the real RAM model registers hold words, can have $+$, $-$, \leq . Instance of APSP: n^2 word reg.

Major open question: faster than $O(n^3)$ or APSP. $\Omega(n^2)$

~~Open~~ Q: if only $+$ and compare on words one at a time. Fredman's FS: $O(n^3 / \log^{1/3} n)$ on real RAM

major question: is APSP $\in n^{3-\epsilon}$?

Either all or none of the following in $n^{3-\epsilon}$:

- 1) APSP
- 2) min (edge) weight triangle
- 3) connectivity.

Th1 (new) word. alg for APSP in $n^3 / 2^{L(n)}$, where $L(n) \gg \Omega(\log n)^{1/2}$

Th2 $\exists \epsilon > 0$ depend on $L(n) \gg \Omega(\log n)^{1/2}$

Recently: det. alg for $L(n) \gg \Omega(\log n)^{1/2}$

Cor and problems above have such times

Th3 term: $\forall n$, \exists non-dets for APSP w/ int weights in $[0, n^4]$ in $n^3 / 2^{L(n)}$ with $L(n) \gg \Omega(\log n)^{1/2}$

Intuition: min-plus matrix product

- 1) min-plus are ACO
- 2) Razborov-Smolensky: 2) eff. but on neg. weights of paths w/ Rappoport's 82 method.

2) Razborov-Smolensky '87: lower ACO (0) circ (\exists eff. standard $D(c)$ of program as $\det(\log s)^{O(d)}$ over \mathbb{F}_2 s.t. $\forall c \in \mathbb{F}_2^d \exists P \in \mathbb{F}_2^{d \times d} = D(c) \exists \exists \exists \exists$

3) polynomial evaluation & FFT: univariate deg- n poly on n roots in \mathbb{C} or \mathbb{F}_2 . In given $A, B \in \{0,1\}^n$, $|A| = |B| = n$, $\{a_1, \dots, a_n, b_1, \dots, b_n\}$ over \mathbb{F}_2 w/ $|B| \leq n^{O(1)}$, can eval g on all $(a_i, b_j) \in A \times B$ in $n^2 \log(\log n)$ time.

Proof: embed in efficient matrix product

APSP alg: given A, B , compute $(A \circ B)[i,j] = \min_k (A[i,k] + B[k,j])$ $\text{lex } d=2$

1) partition A into $n \times d$ matrices $A_{1,1}, \dots, A_{1,d}$, vertical, and B horizontally. Done with over d times $O(n^2 \cdot \frac{n}{d})$ time.

2) given A_k, B_k , $n \times d$ and $d \times n$, let C be ACO circ or min-plus, of degree n words $O(d \log n)$ size. Pick nonzero p_1, \dots, p_{d-1} , eval on rows of A_k , $\log n$ words. $\text{lex } d=2$, c_i kept (2)

see FSTTCS14 survey, AWS SODA15, & a FOCS15

efficient eval \mathbb{C} [14] for $d \approx 2$ min threshold give on combinatorial rectangles. \Rightarrow if min-plus in $d \approx 2$ time, then APSP $\in n^{3-\epsilon}$