

8/9/2015 Russell's Lemma 4

Lemma: if F is depth size S , $d \geq 2 \log_2 S$, circ, $p = 1/D^{d-1}$

Then $\text{Prob} [F|_p \text{ is } \mathcal{O}(D^d)] \leq 2^{-D}$

Cor $\text{Prob} [F|_p \text{ has } d_s \neq 0 \text{ for } d_s > D] \leq 2^{-D}$

Cor $E[\sum_{s, |s| > D} (\hat{d}_s)^2] \leq 2^{-D}$ (w prob. $1 - \frac{1}{2^D}$ $d_s = 0$)

Cor $\sum_{T, |T| > O(D^d)} (\hat{d}_T)^2 \leq O(2^{-D})$

Lemma let f, g be multilinear polynomials over reals,
 $f = \sum_S \beta_S \prod_{i \in S} x_i$, $g = \sum_S \gamma_S \prod_{i \in S} x_i$; then $\sum_x (f(x) - g(x))^2$

$$= \sum_S (\beta_S - \gamma_S)^2 = \|f - g\|^2$$

Set $D = O(\log_2 S + \log_2 1/\epsilon)$, $m = \binom{n}{0(D^d)}$, $S = (\epsilon/2m)^{1/2}$

1) set $\approx 1/d^2 \log_2(m/\epsilon)$ random samples of form $(\vec{x}, P(\vec{x}))$

For each monomial T , $|T| \leq O(D^d)$, use samples to approx

~~total~~ $\text{Prob} [F|_D \text{ is } \mathcal{O}(D^d) - \text{Prob} \neq] \approx \epsilon_T$

$$g = \sum_{|T| > O(D^d)} \prod_{i \in T} x_i$$

$$\sum_{|S| > O(D^d)} (\hat{d}_S - \gamma_S)^2 + \sum_{|S| > O(D^d)} (\hat{d}_S - \gamma_S)^2 \leq \sum_{|S| > O(D^d)} \sigma^2 + 2^{-D} \leq \epsilon$$

since $m D^2 + 2^D \leq \epsilon$

$$\frac{1}{2^n} \sum_{\vec{x}} (f(\vec{x}) - g(\vec{x}))^2 \geq 1, \quad \text{Prob}_x [f(x) \neq h(x)] \leq \epsilon \text{ where}$$

$$x, h(x) \neq f(x) \quad c(\log S / \epsilon)^d$$

$$\text{Time} = \text{poly}(n/\epsilon) = (n/\epsilon)^{c(\log S / \epsilon)^d}$$

Open & proper reading.

Next topic: representation of matrices as low-degree over finite field

\mathbb{R}^2 : θ_1 , every point can be written as $P(x) = \sum_S \beta_S \prod_{i \in S} x_i$

\mathbb{R}^2 : $P \in \mathbb{R}_1$ or $F|_{x_i=1}$

$A \subset \mathbb{R}^2$: depth d constant circ w/ $1, V_i$ \mathbb{O} nodes

$\alpha_i, 1 \leq i \leq n = \prod_{i=1}^n \alpha_i$; no small deg. DR, as well.
 - no small Fourier coeffs, since partition.

$$\oplus \alpha_i = \sum \alpha_i$$

A probability distribution over polynomials

$$\text{Approx } \# \text{ vs } \text{Prob} [f(x) \neq \text{PGI}] \leq \epsilon$$

$\alpha_i, 1 \leq i \leq n$. Let $\epsilon = 1/2$. Pick a subset of $\alpha_i, 1 \leq i \leq n$ at random, then $\sum_{i \in S} \alpha_i$

9/9/2015 Ch. 12, "PPAD"

NP: given $f, f: [n] \rightarrow [n] \exists ? \bar{x} \in [n]^k \Phi(\bar{x}, \bar{y})$

TFNP: complex problems? $\exists \bar{x} \Phi(\bar{x}, \bar{y})$ is valid for all \bar{y} .

Idea: express problems in TFNP in terms of proof strategies (combinatorial lemmas). 5 such lemmas known.

1) Pigeonhole principle: given $f: [n] \rightarrow [n]$, find $x, f(x) = 0$ or $f(x) = f(y), x \neq y$ PPD

2) Parity: given odd deg. hypergraph and coloring PPA

3) PPA \exists : given v_i, v_j and u to be reached node, find x such that $\Phi(x, u)$ PPA

4) PPADS;

5) PLS: given dag w/ nodes solved, find sink PPAD

6) CLS: $\text{PLS} \cup \text{continuous optimization}$ DLS

2) Combinatorial nullstellensatz?

Bertrand postulate?

LLL?

unique solutions?

Ramsey? (given G , find clique or anticlique)

is not hard complete problems for PPA \exists , PLS (not meta-problems)

PPP: Minkowski's theorem

PPA: Smith (if n of hypergraph has even edge), Chevalley mod 2