Continuous-Time Filters

Hakan Kuntman, 2009
A filter is a twoport that shapes the spectrum of the input signal in order to obtain an output signal with the desired frequency content. Passbands where the frequency components are transmitted to the output and stopbands where they are rejected. Traditionally, such circuits working in the continuous-time domain have been designed as resistively terminated lossless LC filters.
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With the growing pressure towards microminiaturisation, inductors were found to be too bulky so that designers started to replace passive RLC filters by active RC circuits where gain, obtained from operational amplifiers (op amps), together with resistors and capacitors in feedback networks, was used to achieve complex poles.

One disadvantage of op amp-based active RC filters is the limited frequency range over which these circuits can be used: the finite bandwidth of op amps usually constrains the applications to be below 100 kHz, with performance deviations becoming increasingly worrisome and difficult to control as operating frequencies increase.
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Other active devices suitable for active filter applications

• Operational Transconductor Amplifier: OTA
• Current Conveyor: CCI, CCII, CCIII, CCCII, DDCCII, DVCCII etc.
• Current-Feedback Operational Amplifier: CFOA
• Current-Differencing Buffered Amplifier: CDBA
• Four-Terminal Floating Nullor: FTFN
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Operational Transconductor Amplifier: OTA,
Dual Output Transconductance Amplifier: DOTA

\[ G = \frac{I_o}{V_{I1}-V_{I2}} \]

input currents \( I_1 = 0, I_2 = 0 \)

\[ G = f(I_A), \text{ controlled by biasing current } I_A \]
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Dual Output Operational Transconductance Amplifier: DOTA

g_{m1} and g_{m2} are controlled by a common biasing current IA

\[ I_o^+ = g_{m1}(V^+ - V^-) \]

\[ I_o^- = g_{m2}(V^- - V^+) \]
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Current-Mode LP Filter

\[ H(s) = \frac{a_0}{s^2 + b_1 s + b_0} \]

\[ \frac{g_{m1}}{C_1} = \frac{b_0}{b_1}, \quad \frac{g_{m2}}{C_2} = b_1, \quad a_0 = b_0 \]

\[ b_1 = \frac{\omega_p}{Q_p}, \quad b_0 = \omega_p^2 \]

\[ \omega_p = \sqrt{\frac{g_{m1} \cdot g_{m2}}{\sqrt{C_1} \cdot \sqrt{C_2}}} \]

\[ Q_p = \sqrt{\frac{g_{m1} \cdot \sqrt{C_2}}{g_{m2} \cdot \sqrt{C_1}}} \]
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Current-Mode BP Filter

\[ H(s) = \frac{a_1 s}{s^2 + b_1 s + b_0} \]

\[ \frac{g_{m1}}{C_1} = \frac{b_0}{b_1}, \quad \frac{g_{m2}}{C_2} = b_1, \quad \frac{g_{m3}}{C_2} = a_1 \]

\[ b_1 = \frac{\omega_p}{Q_p}, \quad b_0 = \omega_p^2 \]

\[ \omega_p = \sqrt{\frac{g_{m1}}{C_1} \cdot \frac{g_{m2}}{C_2}} \quad Q_p = \frac{\sqrt{g_{m1}} \cdot \sqrt{C_2}}{\sqrt{g_{m2}} \cdot \sqrt{C_1}} \]
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Current-Mode HP Filter

\[ H(s) = \frac{a_2 s^2}{s^2 + b_1 s + b_0} \]

\[ \frac{g_{m1}}{C_1} = \frac{b_0}{b_1}, \quad \frac{g_{m3}}{g_{m4}} = a_2, \quad \frac{g_{m2}}{C_2} = \frac{b_2}{a_2} \]

\[ b_1 = \frac{\omega_p}{Q_p}, \quad b_0 = \omega_p^2 \]

\[ \omega_p = \frac{\sqrt{g_{m1} \cdot \sqrt{g_{m2}} \cdot \sqrt{g_{m3}}}}{\sqrt{C_1} \cdot \sqrt{C_2} \cdot \sqrt{g_{m4}}} \]

\[ Q_p = \frac{\sqrt{g_{m1}} \cdot \sqrt{g_{m4}} \cdot \sqrt{C_2}}{\sqrt{g_{m2}} \cdot \sqrt{g_{m3}} \cdot \sqrt{C_1}} \]
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CCII Based Filters

Multi-Input Single-Output Filter
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Setting the currents $i_5 = i_4 = 0$ and applying an input current of $i_2$ to the circuit, a low-pass filter function is obtained which is described by

$$\frac{i_{out}}{i_2} = \frac{G_2 G_3}{s^2 C_1 C_2 + s (C_1 + C_2) G_2 + G_2 G_3}$$

Taking the currents $i_2 = i_5 = 0$ and applying an input current of $i_4$ to the circuit, a band-pass filter function is obtained. The BP transfer function is given by

$$\frac{i_{out}}{i_4} = \frac{s C_1 G_3}{s^2 C_1 C_2 + s G_2 (C_1 + C_2) + G_2 G_3}$$ (3)
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\[ i_2 = -i_5 \]

and

\[ i_4 = i_5 \frac{G_2(C_1 + C_2)}{G_3 C_1} \]

a high-pass filter is obtained. The HP transfer function is described by

\[ \frac{i_{out}}{i_5} = \frac{s^2 C_1 C_2}{s^2 C_1 C_2 + s G_2 (C_1 + C_2) + G_2 G_3} \]
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The pole angular frequency and the quality factor of the filter are given by

$$\omega_0 = \sqrt{\frac{G_2 G_3}{C_1 C_2}}$$

$$Q = \frac{1}{(C_1 + C_2) \sqrt{\frac{C_1 C_2 G_3}{G_2}}}$$

Interchanging resistors and capacitors (RC transformation) the low-pass filter converts to high-pass filter without any component matching
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CCII Based Filters
Realization of Current-Mode Third Order Butterworth Filters
Employing Equal Valued Passive Elements and Unity Gain Buffers

Fig. 1. Third order Butterworth filters with all equal valued capacitors and resistors.
Continuous-Time Filters

The current transfer function of the circuit in Fig. 1(a) is given as

\[
\frac{i_0}{i_{in}} = \frac{sCG^2}{(G + sC)(G^2 + sCG + s^2C^2)}
\]  

(1)

Taking \( G = 1 \) mho and \( C = 1 \) F,

\[
\frac{i_0}{i_{in}} = \frac{s}{(1 + 2s + 2s^2 + s^3)}
\]  

(2)

Impedance and frequency scaling can easily be performed on the filter circuit to shift the center frequency to the desired value.
Performing RC-CR transformation to the circuit which because of symmetry requires only the lower grounded capacitor and resistor to be exchanged another type of band-pass filter function namely

\[
\frac{i_0}{i_{in}} = \frac{s^2}{(1 + 2s + 2s^2 + s^3)}
\]  

(3)

can be implemented. Resulting filter topology is illustrated in Fig. 1(b).
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The core of the circuit consists of two unity voltage gain cells, two capacitors and two resistors and is shown in Fig. 2. The unity gain cells are replaced by second generation current conveyors to pick up the output current. The current transfer functions are given as,
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\[ i_{01} = \frac{sC_2(G_1i_{in1} - sC_1i_{in2})}{\Delta(s)} \]  
\[ i_{02} = \frac{-G_2(G_1i_{in1} - sC_1i_{in2})}{\Delta(s)} \]  

where \( \Delta(s) = s^2C_1C_2 + sG_1C_1 + G_1G_2 \). Thus the core circuit can realize all the three basic filtering functions depending on the selection of the input and output ports.
Fig. 3. Experimental and theoretical frequency responses of proposed third-order Butterworth filters and sixth-order filter obtained by cascading topologies illustrated in Fig. 1(a) and Fig. 1(b) called Cell 1 and Cell 2, respectively.
Continuous-Time Filters

Wide Dynamic Range High Output Impedance Current-mode Multifunction Filters with Dual-output Current Conveyors

Fig. 1. The dual-output current conveyor symbol.

\[
\begin{bmatrix}
v_x \\
v_y \\
v_{z1} \\
v_{z2}
\end{bmatrix} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
\pm 1 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
i_x \\
i_y \\
i_{z1} \\
i_{z2}
\end{bmatrix}
\]

The ± sign indicates whether the element is of p (DO-CCII\text{P}) or n type (DO-CCII\text{N}). This active element has been used in other applications [18, 19]; bipolar [20] and CMOS realisations [21] are presented in literature. In case of non-ideal DO-CCII the definition equation converts to

\[
\begin{bmatrix}
v_x \\
v_y \\
v_{z1} \\
v_{z2}
\end{bmatrix} = \begin{bmatrix}
0 & \beta & 0 & 0 \\
0 & 0 & 0 & 0 \\
\alpha_1 & 0 & 0 & 0 \\
\pm \alpha_2 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
i_x \\
i_y \\
i_{z1} \\
i_{z2}
\end{bmatrix}
\]

where \(\alpha_1, \alpha_2\) denote the current tracking and \(\beta\) denotes the voltage tracking coefficients respectively.
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(c)

(d)

(e)
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\[
H_{HP}(s) = \frac{H_1 s^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2} \tag{2}
\]

\[
H_{LP}(s) = \frac{H_2 \omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2} \tag{3}
\]

\[
H_{BP}(s) = \frac{H_3 \frac{\omega_0}{Q} s}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2} \tag{4}
\]
## Continuous-Time Filters

<table>
<thead>
<tr>
<th>Topology</th>
<th>$\omega_o$</th>
<th>$Q$</th>
<th>$H_1$</th>
<th>$H_2$</th>
<th>$H_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 2a</td>
<td>$\sqrt{\frac{G_1 G_2 G_3}{C_5 C_6 G_4}}$</td>
<td>$\frac{1}{C_1} \sqrt{\frac{G_1 G_4 C_5 C_6}{G_2 G_3}}$</td>
<td>1</td>
<td>$-\frac{G_4}{G_1}$</td>
<td>$\frac{G_4 C_6}{G_2 C_1}$</td>
</tr>
<tr>
<td>Figure 2b</td>
<td>$\sqrt{\frac{G_1 G_2 G_3}{C_1 C_4 G_6}}$</td>
<td>$\frac{1}{C_5} \sqrt{\frac{G_5 G_6 C_1 C_4}{G_2 G_3}}$</td>
<td>1</td>
<td>$\frac{G_6}{G_1}$</td>
<td>$\frac{G_6 C_1}{G_2 C_3}$</td>
</tr>
<tr>
<td>Figure 2c</td>
<td>$\sqrt{\frac{G_1 G_2 G_3}{C_2 C_4}}$</td>
<td>$\frac{1}{C_6} \sqrt{\frac{G_6 C_3 C_4}{G_5}}$</td>
<td>1</td>
<td>$\frac{G_1}{G_6}$</td>
<td>$\frac{C_6}{C_4}$</td>
</tr>
<tr>
<td>Figure 2d</td>
<td>$\sqrt{\frac{G_2 G_3}{C_1 C_4}}$</td>
<td>$\frac{1}{G_1} \sqrt{\frac{G_2 G_3 C_1}{C_4}}$</td>
<td>$-\frac{C_3}{C_1}$</td>
<td>1</td>
<td>$-\frac{G_1}{G_1}$</td>
</tr>
<tr>
<td>Figure 2e</td>
<td>$\sqrt{\frac{G_2 G_3}{C_1 C_5}}$</td>
<td>$\frac{1}{G_1} \sqrt{\frac{G_2 G_3 C_1}{C_5}}$</td>
<td>$-\frac{1}{2} \frac{C_4}{C_1}$</td>
<td>$-1$</td>
<td>$-\frac{G_3}{G_1}$</td>
</tr>
</tbody>
</table>
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The passive sensitivities are calculated for the topology given in Figure 2a as

\[ S_{G_1, G_2, G_3}^{\omega \gamma} = 1/2, \quad S_{G_4, C_5, C_6}^{\omega \gamma} = -1/2, \quad S_{C_1}^{\omega \gamma} = 0, \]

for Figure 2b as,

\[ S_{G_2, G_3}^{\omega \gamma} = 1/2, \quad S_{C_1, C_4, G_6}^{\omega \gamma} = -1/2, \quad S_{C_5}^{\omega \gamma} = 0, \]

\[ S_{C_1, C_4, G_5, G_6}^{\omega \gamma} = 1/2, \quad S_{G_2, G_3}^{\omega \gamma} = -1/2, \quad S_{C_3}^{\omega \gamma} = -1, \]

for Figure 2c as,

\[ S_{G_5, G_6}^{\omega \gamma} = 1/2, \quad S_{C_2, C_4}^{\omega \gamma} = -1/2, \quad S_{G_1, C_6}^{\omega \gamma} = 0, \]

\[ S_{C_2, C_4, G_6}^{\omega \gamma} = 1/2, \quad S_{C_5}^{\omega \gamma} = -1/2, \quad S_{C_6}^{\omega \gamma} = -1, \quad S_{C_1}^{\omega \gamma} = 0, \]

for Figure 2d as,

\[ S_{G_2, G_3}^{\omega \gamma} = 1/2, \quad S_{C_1, C_4}^{\omega \gamma} = -1/2, \quad S_{G_1, C_5}^{\omega \gamma} = 0, \]

\[ S_{C_1, C_4, G_3}^{\omega \gamma} = -1/2, \quad S_{G_4}^{\omega \gamma} = -1/2, \quad S_{C_1}^{\omega \gamma} = -1, \quad S_{C_5}^{\omega \gamma} = 0 \]

and for Figure 2e as,

\[ S_{G_2, G_3}^{\omega \gamma} = 1/2, \quad S_{C_1, C_5}^{\omega \gamma} = -1/2, \quad S_{G_1, C_4}^{\omega \gamma} = 0, \]

\[ S_{C_1, G_2, G_3}^{\omega \gamma} = 1/2, \quad S_{C_5}^{\omega \gamma} = -1/2, \quad S_{C_4}^{\omega \gamma} = 0, \quad S_{C_1}^{\omega \gamma} = -1 \]
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Four Terminal Floating Nullor (FTFN) based filters

An FTFN is equivalent to an ideal nullor or is called operational floating amplifier.

\[ I_1 = I_2 = 0 \quad I_{o1} = I_{o2} \quad V_x = V_y \]

Fig. 1. Nullor model and symbol of FTFN.
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Four Terminal Floating Nullor (FTFN) based filters

FTFN can be considered as a DOTA with infinite transconductance. Practically, DOTA with a very large $g_m$.

As a result:

$I_1 = 0$, $I_2 = 0$,
$V_+ = V_-$
$I_{o1} = I_{o2}$
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Fig. 2. The proposed current-mode multifunction filter.
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\[ \frac{I_{LP}}{I_{in}} = \frac{G_1 G_2}{s^2 C_3 C_5 + s (C_3 + C_5) G_1 + G_1 G_2}, \]

\[ \frac{I_{HP}}{I_{in}} = \frac{-s^2 C_5 C_6}{s^2 C_3 C_5 + s (C_3 + C_5) G_1 + G_1 G_2}, \]

\[ \frac{I_{BP}}{I_{in}} = \frac{-s C_5 G_4}{s^2 C_3 C_5 + s (C_3 + C_5) G_1 + G_1 G_2}. \]

\[ \omega_0 = \sqrt{\frac{G_1 G_2}{C_3 C_5}}, \]

\[ \frac{\omega_0}{Q} = \frac{G_1 (C_3 + C_5)}{C_3 C_5}, \]

\[ Q = \sqrt{\frac{G_2}{G_1} \frac{\sqrt{C_3 C_5}}{C_3 + C_5}}. \]

\[ S_{\omega_0}^{R_1} = S_{\omega_0}^{R_2} = S_{\omega_0}^{C_3} = S_{\omega_0}^{C_5} = -S_R^Q = S_R^Q = -\frac{1}{2}, \]

\[ S_{C_3}^Q = S_{C_5}^Q = \frac{C_5 - C_3}{2 (C_3 + C_5)}. \]
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Current Differencing Buffered Amplifier (CDBA)

The circuit symbol of the CDBA is shown in Figure 1a, where p and n are input, w and z are output terminals. The equivalent circuit of the CDBA is given in Figure 1b. The current differencing buffered amplifier is characterized by Eq. (1).

\[
\begin{bmatrix}
    i_z \\
    v_w \\
    v_p \\
    v_n \\
\end{bmatrix} =
\begin{bmatrix}
    0 & 0 & 1 & -1 \\
    1 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
    v_z \\
    i_w \\
    i_p \\
    i_n \\
\end{bmatrix}
\]  

(1)

Fig. 1. a) Circuit symbol of CDBA. b) Equivalent circuit of CDBA.
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Fig. 2. The proposed filter topology.

\[
\frac{i_{o1}}{i_{in}} = \frac{\frac{G_2}{C_2} s}{s^2 + \frac{G_1+G_2+G_3}{C_1} s + \frac{G_1 G_2}{C_1 C_2}}
\]

(2)

\[
\frac{i_{o2}}{i_{in}} = \frac{\frac{G_2 G_3}{C_1 C_2}}{s^2 + \frac{G_1+G_2+G_3}{C_1} s + \frac{G_1 G_2}{C_1 C_2}}
\]

(3)

\[
\frac{i_{o3}}{i_{in}} = \frac{s^2 + \frac{G_1+G_2+G_3}{C_1} s + \frac{G_1 G_2}{C_1 C_2}}{s^2 + \frac{G_1+G_2+G_3}{C_1} s + \frac{G_1 G_2}{C_1 C_2}}
\]

(4)
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\[ \omega_p = \sqrt{\frac{G_1 G_2}{C_1 C_2}} \]  \hspace{1cm} \text{(5)}

\[ Q_p = \frac{1}{G_1 + G_2 + G_3} \sqrt{\frac{G_1 G_2 C_1}{C_2}} \]  \hspace{1cm} \text{(6)}

\[ S_{G_1}^{op} = S_{G_2}^{op} = 1/2 \quad S_{G_3}^{op} = 0 \]

\[ S_{C_1}^{op} = S_{C_2}^{op} = -1/2 \]

\[ S_{Q_1}^{op} = \frac{G_2 - G_1 + G_3}{2(G_1 + G_2 + G_3)} \]

\[ S_{Q_2}^{op} = \frac{G_1 - G_2 + G_3}{2(G_1 + G_2 + G_3)} \]

\[ S_{Q_3}^{op} = -\frac{G_3}{G_1 + G_2 + G_3} \]

\[ S_{C_1}^{op} = 1/2 \]

\[ S_{C_2}^{op} = -1/2 \]
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Differential Voltage Current Conveyors: DVCC

The DVCC has the advantages of both of the second generation current conveyor (CCII) (such as large signal bandwidth, great linearity, wide dynamic range) and the differential difference amplifier (DDA) (such as high input impedance and arithmetic operation capability).
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**Figure 5.** The basic signal processing block of implementing CM KHN-biquad

(a)

(b)
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\[
\begin{align*}
I_{\text{HP}} &= \frac{(\frac{R_1}{R_2})s^2}{s^2 + (1/C_1R_2)s + (\frac{R_1}{R_2R_3R_4C_1C_2})}, \\
I_{\text{BP}} &= \frac{(\frac{R_1}{R_2R_3C_1})s}{s^2 + (1/C_1R_2)s + (\frac{R_1}{R_2R_3R_4C_1C_2})}, \\
I_{\text{LP}} &= \frac{(\frac{R_1}{R_2R_3R_4C_1C_2})}{s^2 + (1/C_1R_2)s + (\frac{R_1}{R_2R_3R_4C_1C_2})}.
\end{align*}
\] (4a, 4b, 4c)

The natural angular frequency \( \omega_0 \) and the quality factor \( Q \) of the filter can be expressed as

\[
\omega_0 = \sqrt{\frac{R_1}{R_2R_3R_4C_1C_2}}, \quad Q = \sqrt{\frac{R_1R_2C_1}{R_3R_4C_2}}.
\] (5)
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Current Differencing Transconductance Amplifier : CDTA

$V_p = V_n = 0, \quad I_z = I_p - I_n,$

$I_{x+} = gV_z, \quad I_{x-} = -gV_z.$

Current differencing transconductance amplifier (CDTA) is a recently reported current-mode active building block.

Fig. 1. Symbol of the CDTA element.
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Fig. 8. Seventh-order passive elliptic low-pass filter.

Fig. 9. Seventh-order passive elliptic low-pass filter employing CDTAs.
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Fig. 11. Ideal and simulated filter responses.
References

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