Write the recurrence relation for the function definition to the right.

\[
T(n) = \begin{cases} 
\text{________________________} & \text{if } n \text{ ________} \\
\text{________________________} & \text{if } n \text{ ________} 
\end{cases}
\]

Calculate the first 6 terms of \(T(n)\). Then to the right calculate the sequence of differences between these terms. (You do not necessarily have to use all the slots for the sequences of differences - use what you need.)

\[
\begin{array}{cccc}
\text{n} & \text{Sequences of differences} \\
1 & \text{________________________} = ____ \\
2 & \text{________________________} = ____ \\
3 & \text{________________________} = ____ \\
4 & \text{________________________} = ____ \\
5 & \text{________________________} = ____ \\
6 & \text{________________________} = ____ \\
\end{array}
\]

Based on the sequence of differences (above) what is a good guess for the closed-form solution to the recurrence relation above?

\[
f(n) = \text{___________} \\
\]

A) \(n + 2n\) \quad C) \(n^3 + n\) \quad E) \(n^4 + n^2\)
B) \(2^n + n\) \quad D) \(2n + n\) \quad F) \(n^2 + n\)

Why?:

Verify this with a proof by induction. Prove \(T(n) = f(n)\) for all \(n \text{ _________} \).

\[
\begin{align*}
\text{Proof (Induction on } n) & : \text{ If } n = \___, \text{ the recurrence relation says } T(\___) = \___, \\
& \text{ and the closed-form solution says } f(\___) = \text{___________} = \___, \text{ so } T(\___) = f(\___). \\
\text{Suppose as inductive hypothesis that } & T(k-1) = \text{___________} \text{ for some } k > \___. \\
\text{Using the recurrence relation, } & T(k) = \text{___________}, \text{ by } 2^{\text{nd}} \text{ part of RR} \\
& \text{___________}, \text{ by IHOP} \\
& \text{___________} \\
\end{align*}
\]

So, by induction, \(T(n) = f(n) = \text{___________} \text{ for all } n \text{ _________} \) (as desired).
Which of the general types of decompositions of recursive algorithms discussed in class would be good to use for each of the following?

Calculate the factorial of $n$ as a recursive computer program taking a single formal parameter

Reverse chars in an array in place

Check if a string is a palindrome

Build a fractal like Koch snowflake

Binary search and merge sort

Reverse a string by moving last element to front, recurse on rest $(sa)^R \rightarrow a(s)^R$

Suppose that today (year 0) your car is worth $24,000. Each year your car loses 4% of its value, but at the end of each year you add customizations to your car which increase its value by $80. Write a recurrence relation to model this situation.

$$C(n) = \begin{cases} \text{________________________} & \text{if } n \text{ ________} \\ \text{________________________} & \text{if } n \text{ ________} \end{cases}$$

Given the recursive definition:

$B$: $0 \in E$.
$R$: If $n \in E$ so are $n + 2$ and $n - 2$.

which of the following accurately describes what this defines? _____

1) The set of positive odd integers
2) The set of negative odd integers
3) The set of odd integers (including both positive and negative odd integers)
4) The set of odd integers (including both positive and negative odd integers and also zero)
5) The set of positive even integers
6) The set of negative even integers
7) The set of even integers (including both positive and negative even integers)
8) The set of even integers (including both positive and negative even integers, but not zero)

Match the person to what the person is famous for.

_____ Helped break the German Enigma cipher.
_____ Proved that what is now known as the halting problem is undecidable.
_____ Merge sort algorithm.
_____ Credited as being the founding father of Computer Science as a separate discipline.
_____ Described early single-memory, stored program architecture is now commonly known as the general purpose computer.

1) Alan Perlis
2) Alan Turing
3) John von Neumann