Calculate the first 6 terms for the sum of the first $n$ odd natural numbers ($n = 1, 2, 3, \ldots, 6$). Then to the right calculate the sequence of differences between these terms.

<table>
<thead>
<tr>
<th>(n)</th>
<th>Sequences of differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>_______________________ = _____</td>
</tr>
<tr>
<td>2</td>
<td>_______________________ = _____   _____</td>
</tr>
<tr>
<td>3</td>
<td>_______________________ = _____   _____</td>
</tr>
<tr>
<td>4</td>
<td>_______________________ = _____   _____</td>
</tr>
<tr>
<td>5</td>
<td>_______________________ = _____   _____</td>
</tr>
<tr>
<td>6</td>
<td>_______________________ = _____</td>
</tr>
</tbody>
</table>

Write the recurrence relation for the sum of the first $n$ odd natural numbers $S(n)$.

\[
S(n) = \begin{cases} 
\quad \text{if } n \ ____, \ \text{if } n \ __\end{cases}
\]

What does the sequence of differences (above) suggest for the closed-form solution to the recurrence relation?

What is a good guess for the closed-form solution to the recurrence relation above?

\[
f(n) = \ __\]

Verify this with a proof by induction. Prove $S(n) = f(n)$ for all $n \ __\$.

**Proof (Induction on $n$):**

\[
\ __\ : \text{If } n = \ __, \text{ the recurrence relation says } S(\__) = \ __, \\
\quad \text{and the closed-form solution says } f(\__) = \ __ = \ __, \text{ so } S(\__) = f(\__).
\]

**Inductive Hypothesis:** Suppose as inductive hypothesis that $S(k-1) = \ __$ for some $k > \ __$.

\[
\ __\ : \text{Using the recurrence relation, } S(k) = \ __, \text{ by } \text{2}^\text{nd} \text{ part of RR} \\
\quad = \ __, \text{ by IHOP} \\
\quad = \ __ \\
\quad = \ __
\]

So, by induction, $S(n) = \ __$ for all $n \geq 1$ (as \ __\).
What are the four general decompositions of recursive algorithms discussed in class?

_____________________________  _______________________________

_____________________________  _______________________________

Every year, Alice gets a raise of $2,500 plus 4% of her previous year's salary. Her starting salary is $35,000. Give a recurrence relation for $S(n)$, Alice's salary after $n$ years, for all $n \geq 0$.

$$S(n) = \begin{cases} \text{________________________________} & \text{if } n \text{ ______} \\ \text{________________________________} & \text{if } n \text{ ______} \end{cases}$$

List three things Alan Turing is famous for:

________________________________

________________________________

________________________________