Write the recurrence relation for the function definition to the right.

\[ R(n) = \begin{cases} \quad \text{____________________________} & \text{if } n \text{ ________} \\ \quad \text{____________________________} & \text{if } n \text{ ________} \end{cases} \]

Calculate the first 6 terms of \( R(n) \). Then to the right calculate the sequence of differences between these terms. (You do not necessarily have to use all the slots for the sequences of differences - use what you need.)

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c|c|c|c}
\hline
n & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline
\text{Sequences of differences} & & & & & & \\
0 & \quad \text{____________________________} & \quad = & \quad \text{____} \\ 1 & \quad \text{____________________________} & \quad = & \quad \text{____} & \quad = & \quad \text{____} \\ 2 & \quad \text{____________________________} & \quad = & \quad \text{____} & \quad = & \quad \text{____} & \quad = & \quad \text{____} \\ 3 & \quad \text{____________________________} & \quad = & \quad \text{____} & \quad = & \quad \text{____} & \quad = & \quad \text{____} \\ 4 & \quad \text{____________________________} & \quad = & \quad \text{____} & \quad = & \quad \text{____} \\ 5 & \quad \text{____________________________} & \quad = & \quad \text{____} \\
\end{array}
\]

Based on the sequence of differences (above) what is a good guess for the closed-form solution to the recurrence relation above?

\[ f(n) = \quad \text{__________} \]

A) \( n^2 + 1 \)  \quad C) \( 2n + n + 1 \)  \quad E) \( 2^n + n + 1 \)
B) \( 2n + 1 \)  \quad D) \( n^2 + n + 1 \)  \quad F) \( n^3 + 1 \)

Why?:

Verify this with a proof by induction. Prove \( R(n) = f(n) \) for all \( n \) \__________ .

Proof (Induction on \( n \)):

\begin{enumerate}
\item \textbf{Base Case:} If \( n = \) \______, the recurrence relation says \( R(\)______) = \______. \\
and the closed-form solution says \( f(\)______) = \__________ = \______, so \( R(\)______) = f(\)______).
\item \textbf{Inductive Step:} Suppose as inductive hypothesis that \( R(k-1) = \______________ \) for some \( k \) > \______.
\item \textbf{Inductive Hypothesis:} Using the recurrence relation, \( R(k) = \______________ \), by 2\textsuperscript{nd} part of RR
\[ = \______________ , \text{ by IHOP} \]
\[ = \______________ \\
\[ = \______________ \]
\end{enumerate}

So, by induction, \( R(n) = f(n) = \______________ \) for all \( n \) \__________ (as desired).
Which of the general types of decompositions of recursive algorithms discussed in class would be good to use for each of the following?

Build a fractal like Koch snowflake _______________________________

Binary search and merge sort _______________________________

Check if a string is a palindrome _______________________________

Calculate the factorial of $n$ as a recursive computer program taking a single formal parameter _______________________________

Reverse chars in an array in place _______________________________

Reverse a string by moving last element to front, recurse on rest $(sa)^R \rightarrow a(s)^R$ _______________________________

Coins can be packed into the shape of an equilateral triangle. Let $T(n)$ be the number of coins needed to form a triangle with $n$ coins on each edge.

$T(1) = 1$, there is 1 coin on each edge for a total of 1 coin.
   (not much of a triangle but we have to start somewhere).

$T(2) = 3$, there are 2 coins on each edge of an equilateral triangle for a total of 3 coins.

$T(3) = 6$, there are 3 coins on each edge of an equilateral triangle for a total of 6 coins.

$T(4) = ?$

Give a recurrence relation for $T(n)$, the number of coins needed to form an equilateral triangle shape with $n$ coins on each edge, for $n \geq 1$.

$T(n) = \begin{cases} \text{___________________________} & \text{if } n \text{ ________} \\ \text{___________________________} & \text{if } n \text{ ________} \end{cases}$

Match the person to what the person is famous for.

_____ Merge sort algorithm

A) John von Neumann

B) Alan Perlis

C) Alan Turing

_____ First Turing Award winner

_____ Proposed early stored-program (general purpose) digital computer whose architecture named after him

_____ Credited as being the father of Computer Science

_____ Test to determine a machine's ability to exhibit intelligent behavior similar to an actual human