Midterm
CSE 21
Spring 2010

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(93 points = 100%)
Calculate the first 6 terms for the sum of the first \( n \) even natural numbers \((n = 1, n = 2, n = 3, \ldots, n = 6)\). Then to the right calculate the sequence of differences between these terms. (The first even natural number is 2.)

\[
\begin{array}{c|c|c|c}
\hline
n & \text{Sequences of differences} \\
\hline
1 & \underline{\hspace{3cm}} = \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\
2 & \underline{\hspace{3cm}} = \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\
3 & \underline{\hspace{3cm}} = \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\
4 & \underline{\hspace{3cm}} = \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\
5 & \underline{\hspace{3cm}} = \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\
6 & \underline{\hspace{3cm}} = \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\
\hline
\end{array}
\]

Write the recurrence relation for the sum of the first \( n \) even natural numbers \( S(n) \).

\[
S(n) = \begin{cases} \\ 
\underline{\hspace{3cm}} & \text{if } n \underline{\hspace{1cm}} \\
\underline{\hspace{3cm}} & \text{if } n \underline{\hspace{1cm}} \\
\end{cases}
\]

Based on the sequence of differences (above) what is a good guess for the closed-form solution to the recurrence relation above?

\[
f(n) = \underline{\hspace{3cm}}
\]

\[
\begin{array}{|c|c|c|}
\hline
\text{A)} & 4n - 2n & \text{B)} n^3 - n \\
\hline
\text{C)} & n^4 - n^2 + 2 & \text{D)} n^2 + n \\
\hline
\text{E)} & 2n + n & \text{F)} 2^n + n \\
\hline
\end{array}
\]

**Why?:**

Verify this with a proof by induction. Prove \( S(n) = f(n) \) for all \( n \) ________.  

**Proof (Induction on \( n \)):**

\[
\begin{align*}
\underline{\hspace{3cm}} & : \text{If } n = \underline{\hspace{1cm}} , \text{ the recurrence relation says } S(\underline{\hspace{1cm}}) = \underline{\hspace{1cm}} , \\
& \quad \text{and the closed-form solution says } f(\underline{\hspace{1cm}}) = \underline{\hspace{1cm}} = \underline{\hspace{1cm}}, \text{ so } S(\underline{\hspace{1cm}}) = f(\underline{\hspace{1cm}}). \\
\underline{\hspace{3cm}} & : \text{Suppose as inductive hypothesis that } S(k-1) = \underline{\hspace{3cm}} \text{ for some } k > \underline{\hspace{1cm}}. \\
\underline{\hspace{3cm}} & : \text{Using the recurrence relation, } S(k) = \underline{\hspace{3cm}}, \text{ by } 2^{\text{nd}} \text{ part of RR} \\
& \quad = \underline{\hspace{3cm}}, \text{ by IHOP} \\
& \quad = \underline{\hspace{3cm}} \\
\end{align*}
\]

So, by induction, \( S(n) = \underline{\hspace{3cm}} \) for all \( n \geq 1 \) (as \( \underline{\hspace{1cm}} \)).
What are the four general decompositions of recursive algorithms discussed in class?

Every year, Alice's car depreciates (loses its value) by 5% of its value, but she also pimps out her ride each year which increases its value by $550. The car was worth $20,000 when she got it (year 0). Give a recurrence relation for \( S(n) \), the value of Alice's car after \( n \) years, for all \( n \geq 0 \).

\[
S(n) = \begin{cases} 
\text{_____________________________} & \text{if } n \text{ ________} \\
\text{_____________________________} & \text{if } n \text{ ________} 
\end{cases}
\]

How many different strings can be made from the letters in SLEEPLESSNESS, using all the letters?

How many different strings of length 10 can be formed from a set of 26 refrigerator magnets A-Z?

How many different strings of length 10 can be formed from a 26-character alphabet A-Z where the characters can be repeated/reused?

A certain algorithm processes a list of \( n \) elements. Suppose that \text{Subroutine}_a \text{ requires } n^2 + 2n \text{ operations and } \text{Subroutine}_b \text{ requires } 3n^3 + 7 \text{ operations. Give a big-theta estimate for the number of operations performed by the following pseudocode segment. } \]

\[
\text{for } i \in \{1, 2, \ldots, n\} \text{ do} \\
\quad \text{Subroutine}_a \\
\quad \text{Subroutine}_b
\]

Find a big-theta estimate for each function using an estimation target.

\[
(12n + 17)^{23} \quad n \log_2 n + n! \quad n \log_2 n + n \quad \text{__________} 
\]

Big-oh provides a(n) ________ bound on the growth rate of a function while big-omega provides a(n) ________ bound on the growth rate of a function.
Rank the different time complexity classes from smallest to largest according to how fast they grow as their input $n$ grows large.

_____ smallest/slowest growing (faster algorithms)

A) linear
B) exponential
C) logarithmic
D) factorial
E) $n \log_2 n$
F) constant
G) polynomial

_____ fastest growing (slower algorithms)

How many 5-digit zip codes can be formed with at least one duplicate digit (for example, 00489, 58868, and 33997)? Zip codes can range from all zeros to all nines.

There are thousands of people at sun god. Show there are at least two people at the festival with the same first and last initials. Name the principle used in your explanation. [You cannot say you know Madiha and her sister Maria (or anyone else) will be there and they have the same first and last initials.]

A test has four multiple choices questions. There are three possible answers for each question. There is only one correct answer for each question.

How many different ways are there to fill out the answer sheet?

How many ways are there to fill out the answer so that three answers that are correct and one is incorrect?

Ann guesses randomly since she didn't study. What is the probability that she gets two or fewer questions correct?
Match the person to what the person is famous for.

_____ Known as the father of algorithms.
_____ Hates goto statements.
_____ A pioneer in computer-based random number generators.
_____ Computing's highest honor (Nobel Prize of computing) named after.
_____ Recipient of a Turing Award, John von Neumann Medal, Kyoto Prize, Grace Hopper Award, and others.
_____ Also a recipient of a Turing Award.
_____ A main contributor to cracking the Enigma machine.
_____ Associated with machine-independent programming languages we take for granted today.
_____ Computing's highest honor (Nobel Prize of computing) named after.
_____ Co-authored *Concrete Mathematics* (a blend of CONtinuous and disCRETE math) with Ron Graham.
_____ Allegedly thinks Object-Oriented Programming (OOP) is a bad idea.
_____ Known as the father of theoretical computer science and artificial intelligence.
_____ Helped popularize the term "debugging."
_____ Shortest-Path algorithm.
_____ "Can machines think?" test named after.
_____ A principle developer of ALGOL
_____ Has a well-known conference primarily for women in computing named after.
_____ A theoretical device representing a computing machine to understand limits of computation named after.
_____ Invented the semaphore concept.
_____ Known as the father of the modern computer.
_____ Invented merge sort algorithm.
_____ Prosecuted as a homosexual with a posthumous apology.

A) Edsger Dijkstra
B) Donald Knuth
C) Alan Turing
D) Grace Hopper
E) John von Neumann

Consider the following algorithm:
```
char alpha[] = "ABCDEFGHIJKLMNOPQRSTUVWXYZ";
for ( int i = 0; i < m; ++i )
    {for ( int j = 0; j < n; ++j )
       {cout << alpha[i] << alpha[j] << alpha[i];
        }
    }
```
How many characters are output in terms of m and n?

_______ (in terms of m and n)

Consider the following algorithm:
```
for ( int i = 0; i < n - 1; ++i )
    {for ( int j = i + 1; j < n; ++j )
       {if ( array[i] == array[j] )
          {++numOfDuplicates;
           }
       }
    }
```
How many "==" comparisons are made if n is 10?

_______ (give an exact number answer)
Anu's Café Discrete serves 6 kinds of soups, 5 kinds of salads, and 20 kinds of entrees.

Rick goes to the café on Monday. How many ways can he order a salad or a soup?

Madiha goes to the café on Tuesday. How many ways can she order a salad and a soup?

The CSE21 team (Anu, Brina, Chung, Ko, Madiha, Mohammad) go to the café on Wednesday. They decide to order and share three different entrees among themselves. How many ways can they place such an order?

The CSE21 team (Anu, Brina, Chung, Ko, Madiha, Mohammad) go to the café on Thursday. They decide to order and share three entrees among themselves. How many ways can they place such an order? (Note: All three entrees can be the same entrée.)

The CSE21 team (Anu, Brina, Chung, Ko, Madiha, Mohammad) and Rick go to the café on Friday. Each of them orders their own entrée without regard to what others ordered. How many ways can they place such an order? (Note: Total of seven entrée orders; an entrée may be ordered more than once.)

The CSE21 team (Anu, Brina, Chung, Ko, Madiha, Mohammad) and Rick go to the café on Saturday. There is a waiting line to be seated. How many ways can the seven of them stand single file in line?

When they finally get seated, each of them orders an entrée. However, no two of them order the same entrée. How many ways can they place such an order?
The management decides to have a buffet every day where each of the 20 entrées is served at least once during the week. What is the minimum number of entrées to be served each day of the week to do this?

The management decides to have a soup promotion. Basically, they make any vegetable soup (pumpkin, cauliflower, potato, tomato) with any spice (ginger, garlic, cinnamon, nutmeg). Anu does not like the taste of nutmeg and tomato together. She does not like garlic at all. How many choices does she have (draw a decision tree to help her). Use the first two letters of each soup and spice to fill in the decision tree. List the soups first. Not all slots need to be filled.

```
___                                     ___                                      ___                                       ___
___    ___    ___    ___        ___    ___    ___    ___        ___    ___    ___    ___        ___    ___    ___    ___

How many choices does she have? _____

One fine day, the CSE21 team (6 members) decides to order ten identical entrées (ten of the same entrée). How many ways can all ten entrées be placed in front of the team members (some team members may not get any entrées placed in front of them – for example, all entrées might be placed in front of just Anu!)? Hint: Think bones and dogs.

On Monday, Café Discrete had 32 customers for breakfast and 56 customers for lunch. None of them were repeat customers. How many different customers did they have on Monday?

On Tuesday, the CSE team (6 members) had breakfast and lunch at Café Discrete. In total, the Café had 32 customers for breakfast and 56 customers for lunch. How many different customers did they have on Tuesday?
Scratch Paper