Midterm
CSE 21
Fall 2010

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Page 2  ___________  (10 points)
Page 3  ___________  (18 points)
Page 4  ___________  (11 points)
Page 5  ___________  (8 points)
Page 6  ___________  (5 points)
Total  ___________  (81 points)

(77 points = 100%)
( 4 points Extra Credit)

This exam is to be taken by yourself with closed books, closed notes, no electronic devices. You are allowed one side of an 8.5”x11” sheet of paper handwritten by you.
Consider the following recurrence relation:

\[ S(n) = \begin{cases} 
1 & \text{if } n = 0 \\
S(n - 1) + 2n - 1 & \text{if } n > 0
\end{cases} \]

Calculate the first 6 terms for this recurrence relation \((n = 0, n = 1, n = 2, \ldots, n = 5)\). Then to the right calculate the sequence of differences between these terms.

<table>
<thead>
<tr>
<th>(n)</th>
<th>_______________________</th>
<th>Sequences of differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>_______________________</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>_______________________</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>_______________________</td>
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<tr>
<td>3</td>
<td>_______________________</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>_______________________</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>_______________________</td>
<td></td>
</tr>
</tbody>
</table>

Based on the sequence of differences (above) what is a good guess for the closed-form solution to the recurrence relation above?

\[ f(n) = \]  

Why?:

Verify this with a proof by induction. Prove \(S(n) = f(n)\) for all \(n \) ________ .

Proof (Induction on \(n\)):

\[ \text{If } n = \text{______, the recurrence relation says } S(\text{______}) = \text{______,} \]

and the closed-form solution says \(f(\text{______}) = \text{______} = \text{______, so } S(\text{______}) = f(\text{______}). \)

\[ \text{Suppose as inductive hypothesis that } S(\text{______}) = \text{____________________} \text{ for some } k > \text{______.} \]

\[ \text{Using the recurrence relation, } S(\text{______}) = \text{___________________________}, \text{ by } 2^{\text{nd}} \text{ part of RR} \]

\[ = \text{___________________________}, \text{ by IHOP} \]

\[ = \text{___________________________} \]

So, by induction, \(S(n) = \text{__________} \text{ for all } \text{__________} \) (as \text{__________}).
What are the four general decompositions of recursive algorithms discussed in class?

___________________________________________

___________________________________________

___________________________________________

The Fibonacci numbers $Fib(n)$ satisfy the following recurrence relation:

$$Fib(n) = \begin{cases} 
1 & \text{if } n = 1 \text{ or } n = 2 \\
Fib(n-1) + Fib(n-2) & \text{if } n > 2 
\end{cases}$$

Write a recursive method that returns the $n^{th}$ Fibonacci number. Assume $n \geq 1$. Just do a direct translation.

```java
long Fib( int n )
{
}
```

How many different strings can be made from the letters in ABRACADABRA, using all the letters?

___________________________________________

How many strings of length 6 can be formed from an 11-symbol alphabet where no 2 adjacent symbols are the same?

___________________________________________

How many different strings of length 9 can be formed from a set of 26 refrigerator magnets A-Z?

___________________________________________

How many (nonempty) strings of at most length 3 can be formed from a 26-symbol alphabet?

___________________________________________
Find a big-theta estimate for each function using an estimation target.

\[(12n^3 + 17n^2)^{23} \quad \text{________} \quad n \log_2 n + n! \quad \text{________} \quad n \log_2 n + 32^{10} n \quad \text{________} \]

A certain algorithm processes a list of \(n\) elements. Suppose that \(\text{Subroutine}_a\) requires \(8n^4 + 2n\) operations and \(\text{Subroutine}_b\) requires \(3n^2 + 7\) operations. Give a big-theta estimate for the number of operations performed by the following pseudocode segment. \(\text{________} \)

```plaintext
for i \in \{1, 2, \ldots, n\} do
    \{ Subroutine\textsubscript{a} Subroutine\textsubscript{b} \}
```

Big-Oh provides a(n) \(\text{________}\) bound on the growth rate of a function while big-Omega provides a(n) \(\text{________}\) bound on the growth rate of a function.

Given \(K_1f(n) \leq g(n) \leq K_2f(n)\), we say that "\(g\) is big-\(\text{________}\) of \(f\)" or "\(g\) is order \(f\)."

\(K_1f(n)\) represents the big-\(\text{________}\) of \(f\) while \(K_2f(n)\) represents the big-\(\text{________}\) of \(f\).

With respect to the graph to the right,

the function labeled ____ represents big-Oh of \(f\) and

the function labeled ____ represents big-Omega of \(f\).

Rank the different time complexity classes from smallest to largest according to how fast they grow as their input \(n\) grows large.

______ smallest/slowest growing (faster algorithms)

______

______

______

______

______

______ fastest growing (slower algorithms)
How many 5-digit zip codes are possible? Zip codes can contain all zeros.

How many 5-digit zip codes contain only even digits? 0 is an even number.

How many 5-digit zip codes have at least one odd digit?

How many 5-digit zip codes have all different digits (no duplicates)?

How many 5-digit zip codes start with an even digit and end with an even digit?

How many 5-digit zip codes can be formed with at least one duplicate digit (for example, 00489, 58868, and 33997)?

What is the value of C(7,3)? Your answer should be an actual number for this one.

What is the value of P(8,2)? Your answer should be an actual number for this one.

How many different ways are there to distribute 12 identical bones among 4 different dogs? (No dog can get a negative number of bones.)

How many ways are there to arrange all the letters in the word ABRUPTLY?

What is the probability of rolling a 4 or more (sum of two fair 6-sided dice will be 4 or more)?
A certain board game uses tokens made of transparent colored plastic. Each token looks like where each of the 4 different regions is a different color: either red, green, yellow, blue, orange, or purple. How many different tokens of this type are possible?

How many four-digit binary strings are there that do not contain 100 or 011? First draw a decision tree.

In Craps, Eight the Hardway bet wins if the Hard Eight (4-4) is rolled before a 7 is rolled or before an "easy" eight (2-6, 3-5, 5-3, 6-2) is rolled. All other dice combinations are not considered (you neither win nor lose if any other dice combination is rolled). So this is not a single-roll wager. The only dice combinations that are part of this bet (the set that comprises the Universe in this bet) are those that add up to 7 or 8.

What is the probability of rolling a Hard Eight before (verses) a 7 or an "easy" eight?

\[ P(X=\text{Hard Eight}) = \] 

If the payout for hitting a Hard Eight is 9-to-1 (for example, $1 bet pays $9 + the original $1 bet for a total of $10), the Expected Value of the amount of money you will win in terms of \( P(X=x) \) is

\[ E(X) = 10 * P(X=\text{Hard Eight}) + 0 * P(X = 7 \text{ or } \text{"easy" eight}) \]

Now replace the \( P(X=x) \) values with their numeric probabilities keeping your answer in terms of fractions vs. decimals. Reduced fractions are preferred.

\[ E(X) = 10 * \frac{\text{probability of Hard Eight}}{\text{total probability}} + 0 * \frac{\text{probability of 7 or easy eight}}{\text{total probability}} = \]

If your bet is $1 (costs you $1 to play), what is your expected return each time you make this kind of bet? Express your answer as a positive or negative reduced fraction.

\[ E(X) - 1 = \]
Match the person to what the person is famous for. (1/2 point each)

_____ Known as the father of algorithms.

_____ Computing's highest honor (Nobel Prize of computing) named after.

_____ Helped popularize the term "debugging."

_____ Shortest-Path algorithm.

_____ Credited with inventing Merge sort algorithm.

_____ Co-authored *Concrete Mathematics* (a blend of CONtinuous and disCRETE math) with Ron Graham.

_____ Has a well-known conference primarily for women in computing named after.

_____ A theoretical device representing a computing machine to understand limits of computation named after.

_____ Invented the semaphore concept.

_____ Known as the father of the modern general purpose sequential computer (not the father of modern Computer Science).

1) Edsger Dijkstra
2) Donald Knuth
3) Alan Turing
4) Grace Hopper
5) John von Neumann