Student ID __________________
Name ______________________
Signature _______________________

By filling in the above and signing my name, I confirm I will complete this exam with the utmost integrity and in accordance with the Policy on Integrity of Scholarship.

Final
CSE 21
Spring 2013

Page 1  ___________  (11 points)
Page 2  ___________  (19 points)
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Page 12  ___________  (7 points) Extra Credit

Total  ___________

This exam is to be taken by yourself with closed books, closed notes, no electronic devices. You are allowed both sides of an 8.5"x11" sheet of paper handwritten by you.
Write the recurrence relation for the function definition to the right.

\[ R(n) = \begin{cases} \ \text{___________________________} & \text{if } n \text{ } \underline{\text{______}} \\
\text{___________________________} & \text{if } n \text{ } \underline{\text{______}} \end{cases} \]

Calculate the first 6 terms of \( R(n) \). Then to the right calculate the sequence of differences between these terms. (You do not necessarily have to use all the slots for the sequences of differences - use what you need.)

\[ \begin{array}{llllll}
\text{n} & 0 & 1 & 2 & 3 & 4 & 5 \\
\text{Sequences of differences} & & & & & & \\
0 & \underline{\text{_____________________}} & \underline{\text{_____}} \\
1 & \underline{\text{_____________________}} & \underline{\text{_____}} & \underline{\text{_____}} \\
2 & \underline{\text{_____________________}} & \underline{\text{_____}} & \underline{\text{_____}} & \underline{\text{_____}} & \underline{\text{_____}} \\
3 & \underline{\text{_____________________}} & \underline{\text{_____}} & \underline{\text{_____}} & \underline{\text{_____}} & \underline{\text{_____}} & \underline{\text{_____}} \\
4 & \underline{\text{_____________________}} & \underline{\text{_____}} & \underline{\text{_____}} & \underline{\text{_____}} \\
5 & \underline{\text{_____________________}} & \underline{\text{_____}} & \underline{\text{_____}} \\
\end{array} \]

Based on the sequence of differences (above) what is a good guess for the closed-form solution to the recurrence relation above?

\[ f(n) = \underline{\text{_____________}} \]

Verify this with a proof by induction. Prove \( R(n) = f(n) \) for all \( n \underline{\text{___________}} \).

\textbf{Proof (Induction on } n \text{)}:

\begin{itemize}
  \item \underline{\text{_____________}}: If \( n = \underline{\text{______}} \), the recurrence relation says \( R(\underline{\text{______}}) = \underline{\text{______}} \).
  \item \underline{\text{_____________}}: Suppose as inductive hypothesis that \( R(\underline{\text{______}}) = f(\underline{\text{______}}) \).
  \item \underline{\text{_____________}}: Using the recurrence relation, \( R(k) = \underline{\text{_____________}} \), by 2\textsuperscript{nd} part of RR
    = \underline{\text{_____________}}, by IHOP
    = \underline{\text{_____________}}
    = \underline{\text{_____________}}
\end{itemize}

So, by induction, \( R(n) = f(n) = \underline{\text{_____________}} \) for all \( n \underline{\text{___________}} \) (as desired).
Write the recurrence relation for the sum of the first \( n \) natural numbers.

\[
S(n) = \begin{cases} 
\text{___________________________} & \text{if } n \text{ ______} \\
\text{___________________________} & \text{if } n \text{ ______} 
\end{cases}
\]

Now write the closed-form equation/solution for the sum of the first \( n \) natural numbers (you should know this off the top of your head by now, I hope).

Which of the general types of decompositions of recursive algorithms discussed in class would be good to use for each of the following?

- Binary search and merge sort
- Reverse a string by moving last element to front, recurse on rest \((sa)^R \rightarrow a(s)^R\)
- Calculate the factorial of \( n \) as a recursive computer program taking a single formal parameter
- Reverse chars in an array in place
- Check if a string is a palindrome
- Build a fractal like Koch snowflake

Suppose that you borrow $100 from your friend, Joe. Each \textit{week} you are able to pay him $5, but at the end of each \textit{month} Joe charges you 5% on the amount you still owe. Write a recurrence relation to represent the amount you owe after \( n \) \textit{months}. Assume there are 4 weeks in each month.

\[
C(n) = \begin{cases} 
\text{___________________________} & \text{if } n \text{ ______} \\
\text{___________________________} & \text{if } n \text{ ______} 
\end{cases}
\]

In a class of 11 boys and 9 girls, the teacher selects three students at random to write problems on the board. What is the probability that all the students selected are boys?

If we calculate the probability that all the students selected are girls, which has a greater probability? _____

1) All students selected are boys \hspace{1cm} 2) All students selected are girls \hspace{1cm} 3) They are the same

How many ways are there to rearrange all the letters in NONSENSELESSNESS?

Using two fair 6-sided dice, what is the probability you will roll a 10 or less?
Regarding the List, Sum, SList, Search, BSearch, and Binary Search Trees defined in the textbook …

If List $L = L', x$ for some list $L'$, then

$$\text{Sum1}(L) = \text{Sum1}(L') + x$$

An SList (denoted as $SL$) is $(X, Y)$ where $X$ and $Y$ are SLLists having the same number of elements, and the last number in $X$ is less than the first number in $Y$

$$\text{Search}(t, SL) = \text{Search}(t, X) \lor \text{Search}(t, Y)$$

$$\text{BSearch}(t, SL) = \text{BSearch}(t, Y) \text{ if } t > r \text{ or } \text{BSearch}(t, SL) = \text{BSearch}(t, X) \text{ if } t \leq r \text{ - where } r \text{ is the last element of } X$$

$$\text{Sum2}(SL) = \text{Sum2}(X) + \text{Sum2}(Y)$$

If Binary Search Tree (BST) $T$ has root $r$ and subtrees $T_1$ and $T_2$, then

$$\text{InOrder}(T) = \text{InOrder}(T_1), r, \text{InOrder}(T_2)$$

Which of the following recurrence relations best represents the approximate number of operations required for:

- Sum1($L$) where $n$ is the number of elements in list to be summed? _____
- Search($t, SL$) where $n$ is the number of elements in list to be searched? _____
- BSearch($t, SL$) where $n$ is the number of elements in list to be searched? _____
- Sum2($SL$) where $n$ is the number of elements in list to be summed? _____
- InOrder($T$) where $n$ is the number of nodes in the BST to be listed? _____

1) $T(n) = 2*T(n-1) + 1$
2) $T(n) = T(n-1) + 1$
3) $T(n) = T(n/2) + 1$
4) $T(n) = T(\log_2 n) + 1$
5) $T(n) = 2*T(n-2) + 1$
6) $T(n) = 2*T(n/2) + 1$

If a list/tree contains $n$ elements/nodes,

- Sum1 will require approximately __________ additions(s) to sum the elements in the list.
- Search will require approximately __________ comparison(s) to search for a target in the list.
- BSearch will require approximately __________ comparison(s) to search for a target in the list.
- Sum2 will require approximately __________ additions(s) to sum the elements in the list.
- InOrder will require approximately __________ node visit(s) to list the values in the tree.

How many strings of length 14 can be formed from a set of 26 refrigerator magnets A-Z (no duplicates)?

How many strings of length 17 can be formed from a set of 26-symbol alphabet (duplicates allowed)?

How many strings of length 9 can be formed from a set of 26-symbol alphabet (duplicates allowed) where no two adjacent symbols are the same?
How many UCSD student ids are possible where a student id is made up of a single lower-case alpha character (a-z) followed by 8 digits (0-9)?

How many student ids have all different digits (no duplicates)? [Don't forget the leading char in the following.]

How many student ids contain only odd digits in the digits part of their ids?

How many student ids have at least one even digit?

How many student ids start with an odd digit and end with an even digit in the digits part of their ids?

How many student ids can be formed with at least one duplicate digit (for example, a00456892, d58868312, and y33997971)?

One of Allyson's favorite candy that starts with an S and ends in "ittles" comes in five colors: red, orange, yellow, green, and purple. These are packaged in fun-size bags of 20, but there is no guarantee as to how the colors will be distributed; you might get a mixture of all five colors, or just some red and some green, or even (if you are very lucky) a whole bag of purple. Compute the total number of different possible color distributions.

The school board consists of three men and four women.

a) When they hold a meeting, they sit in a row. How many different seating arrangements are there?

b) How many different ways can the row be arranged if no two women sit next to each other?

c) How many ways are there to select a subcommittee of four board members?

d) How many ways are there to select a subcommittee of four board members if the subcommittee must contain at least two women?
What is a stable sort? _____

A) The sort uses the paradigm of a barn's stable to partition data into different stalls similar to shunting-yard alg.
B) Duplicate values are in the same order after the sort as they were in the original data
C) Duplicate values are eliminated so the sorted data has no duplicates
D) The sort is done in place without using additional space or data structures
E) The sort is always the same runtime complexity no matter what order the original data is in

Prof. N Timmy Date has 30 students in his Programming class and 24 student in his Discrete Math class. Assuming there are eight students who take both classes, how many students does Prof. Date have?

An urn contains seven red balls, seven white balls, and seven blue balls. A sample of five balls is drawn at random without replacement.

a) Compute the probability that the sample contains four balls of one color and one of another color.

b) Compute the probability that all of the balls in the sample are the same color.

What is the value of C(8,4)? Your answer should be an actual number. _______

What is the value of P(8,3)? Your answer should be an actual number. _______

C(26,5) is the same value as C(26, ____ )
P(n,n) = ______ (in terms of n)

C(26,1) is the same value as C(26, ____ ) = ______ (actual #)
P(n,1) = ______ (in terms of n)

C(26,26) is the same value as C(26, ____ ) = ______ (actual #)
P(n,0) = ______

An urn contains 10 balls numbered 1-10. Four balls are drawn from the urn as described below, and the numbers on the balls are recorded. How many ways are there to do this …

… if the four balls are drawn one at a time and not replaced before the next one is drawn?

… if the four balls are drawn all at once?

… if the four balls are drawn one at a time and replaced before the next one is drawn?

In a class of 36, there will always be a group of at least how many who were born on the same day of the week?
Given the binary tree to the right

Specify the output for the following traversals

Preorder traversal: ______________________________

Inorder traversal: ______________________________

Postorder traversal: ______________________________

Breadth-first traversal: ______________________________

Big-Oh provides a(n) ____________ bound on the growth rate of a function while big-Omega provides a(n) ____________ bound on the growth rate of a function.

Given $K_1 f(n) \leq g(n) \leq K_2 f(n)$, we say that "$g$ is big-___________ of $f$" or "$g$ is order $f$.

$K_1 f(n)$ represents the big-___________ of $f$ while $K_2 f(n)$ represents the big-___________ of $f$.

With respect to the graph to the right,

the function labeled _____ represents big-Omega of $f$ and

the function labeled _____ represents big-Oh of $f$.

Consider the following algorithm:

$x \leftarrow 1$

for $i \in \{1, 2, 3, 4\}$ do
  $x \leftarrow x + x$

for $j \in \{1, 2, 3\}$ do
  $x \leftarrow x + 1$

for $k \in \{1, 2, 3, 4\}$ do
  $x \leftarrow x + 5$

Count the number of + operations done by this algorithm. _______
A fair coin is flipped 4 times. Use the decision tree below to fill in the different possibilities with each flip putting an H for Heads on the left and T for Tails on the right. Use this decision tree to calculate the probability that at least 2 consecutive Tails occur during the process of flipping the coin 4 times. Just one H or T per line.

How many sequences of flips contain at least 2 consecutive Tails? _____
What is the probability that at least 2 consecutive Tails occur when flipping the coin 4 times? ______________
What is the probability that at least 3 consecutive Heads occur when flipping the coin 4 times? _______________
What is the probability that the Heads and Tails alternate when flipping the coin 4 times? _________________

If you have an algorithm that when you double the number of elements it …
increases the number of comparisons by 1, this alg. is most likely in what big-Oh complexity class? ____________
quadruples the number of comparisons, this alg. is most likely in what big-Oh complexity class? ____________
it increases the number of comparisons by slightly more than double (2.x times) for sufficiently large n
(say \(n \geq 16\)), this algorithm is most likely in what big-Oh complexity class? ____________
doubles the number of comparisons, this algorithm is most likely in what big-Oh complexity class? ___________
squares the number of comparisons, this algorithm is most likely in what big-Oh complexity class? ___________
does not change the number of comparisons, this alg. is most likely in what big-Oh complexity class? __________

If you write a program which contains an array initialized with \(n\) random unsorted elements that you sort using
Quicksort and then perform 2048 binary searches on this sorted array, what is the overall run time complexity
of the program in terms of \(n\)? O(_______________________)

What if you quadruple the number of binary searches? O(_______________________)

Given the initial order of ints in an array as: 4, 8, 11, 10, 3, 2, 7, 5, 6 what is the order of the elements after
5 iterations of the selection sort algorithm covered in class and one of the HW exercises?

____  ____  ____  ____  ____  ____  ____  ____  ____
Match the big-Oh complexity class value to the graph. Write the letter associated with the function to the right that describes the big-Oh runtime complexity for that function.

Between which two plot lines would we find
O( n^4 ) - between ____ and ____
O( n log_2 n ) - between ____ and ____

What is the name of the complexity class for each labeled function?
(Not big-Oh – the English word name for the complexity class.)
A) ____________________________
B) ____________________________
C) ____________________________
D) ____________________________
E) ____________________________

Match the algorithms with their recurrence relations and their run time complexities. Use the letters and numbers from the boxes to the right.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Recurrence Relation</th>
<th>Time Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selection Sort</td>
<td>______</td>
<td>______</td>
</tr>
<tr>
<td>Binary Tree Traversal</td>
<td>______</td>
<td>______</td>
</tr>
<tr>
<td>Binary Search</td>
<td>______</td>
<td>______</td>
</tr>
<tr>
<td>Sequential Search</td>
<td>______</td>
<td>______</td>
</tr>
<tr>
<td>Merge Sort</td>
<td>______</td>
<td>______</td>
</tr>
<tr>
<td>Towers of Hanoi</td>
<td>______</td>
<td>______</td>
</tr>
<tr>
<td>Bubble Sort</td>
<td>______</td>
<td>______</td>
</tr>
<tr>
<td>Fibonacci sequence (recursive)</td>
<td>______</td>
<td>______</td>
</tr>
<tr>
<td>Fibonacci sequence (iterative as discussed in class)</td>
<td>______</td>
<td></td>
</tr>
<tr>
<td>Array access</td>
<td>______</td>
<td>______</td>
</tr>
</tbody>
</table>

A) T(n) = T(n/2) + O(1)
B) T(n) = T(n-1) + T(n-2) + O(1)
C) T(n) = T(n-1) + O(1)
D) T(n) = 2 T(n-1) + O(1)
E) T(n) = 2 T(n/2) + O(1)
F) T(n) = 2 T(n-1) + O(n)
G) T(n) = T(n-1) + O(n)
H) T(n) = 2 T(n/2) + O(n)

1) O( n )
2) O( 2^n )
3) O( 1 )
4) O( n^2 )
5) O( n log_2 n )
6) O( log_2 n )
7) O( n! )
According to the Lower Bound Theorem, the best (most efficient) worst case complexity for comparison-based search is Θ(_______________)

According to the Lower Bound Theorem, the best (most efficient) worst case complexity for comparison-based sort is Θ(_______________)

Construct a minimum spanning tree from the following network. Use the grayed network on the right to construct your msp. Hint: 9 vertices so msp should have 8 edges.

![Network Diagram]

What is the total weight of the minimum spanning tree? ______
Is there more than one minimum spanning tree in this graph (yes or no)? ______

_____ is the collection of all problems that can be solved with an algorithm whose complexity is, at most, polynomial.

_____ is the collection of all problems whose solutions can be checked (but not necessarily solved) in polynomial time.

What is Rick's cowboy name? _______________________________________________.

Write the correct way to calculate a mid-point index mid based on index values of lo and hi.

Given the sorted array of ints in an array as: 2, 3, 5, 6, 10, 12, 14, 16, 17, 20, 21, 24, 26, 27, 29, 31 what are the values of lo, mid, and hi indexes for each recursion in the recursive binary search algorithm when searching the entire array for the target value 12?

<table>
<thead>
<tr>
<th>lo</th>
<th>mid</th>
<th>hi</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15</td>
<td>16</td>
</tr>
</tbody>
</table>

- starting min and max indexes are given for you

What is the return value of binarySearch() in this case? ______
Consider the following algorithm:

\[
\begin{align*}
& i \leftarrow 0 \\
& j \leftarrow 0 \\
& \text{while } i < n \text{ do} \\
& \quad \{ \\
& \quad \quad i \leftarrow i + 1 \\
& \quad \quad j \leftarrow j + 2 \\
& \quad \}
\end{align*}
\]

Show that the statement \( j = 2i \) is an invariant for this while loop. Use \( \bar{i} \) and \( \bar{j} \) to denote the values of \( i \) and \( j \) before the body of the while executes and use \( \bar{\bar{i}} \) and \( \bar{\bar{j}} \) to denote the values of \( i \) and \( j \) after execution. To prove that \( j = 2i \) is an invariant, we need to show that

\[ \bar{j} = 2\bar{i} \Rightarrow \bar{\bar{j}} = 2\bar{\bar{i}} \]

A regular expression cannot be written to correctly recognize only valid strings that are palindromes or only valid strings in the form \( a^n b^n \) because regular expressions cannot ______________.

In grep (and vim), what metacharacter is used to match 0 or more of the preceding element? ________

In grep (and vim), what metacharacter is used to match any character? ________

In grep (and vim), what metacharacter is used to match 1 or more of the preceding element? ________

In grep (and vim), what metacharacter is used to specify/match the start of the line? ________

\( S \) is the start symbol. \( a \) and \( b \) are terminal symbols.

\[
\begin{array}{llllll}
1) & S \rightarrow Sab & \quad 2) & S \rightarrow aSb & \quad 3) & S \rightarrow abSab \\
   & S \rightarrow ab & \quad & S \rightarrow ab & \quad & S \rightarrow ab \\
4) & S \rightarrow aSa & \quad 5) & S \rightarrow SbS & \quad 6) & S \rightarrow Sb \\
   & S \rightarrow aba & \quad & S \rightarrow a & \quad & S \rightarrow a \\
7) & S \rightarrow aSa & \quad 8) & S \rightarrow aS_i a & \quad 9) & \text{None of the above} \\
   & S \rightarrow b & \quad & S_i \rightarrow bS_i | b & \quad & \\
\end{array}
\]

Which context-free grammar correctly recognizes only words of the language \( (ab)^n \) for \( n > 0 \)? ______

Which context-free grammar correctly recognizes only words of the language \( a^n ba^n \) for \( n \geq 0 \)? ______

Which context-free grammar correctly recognizes only words of the language \( a^n b^n \) for \( n > 0 \)? ______

Which context-free grammar correctly recognizes only words of the language \( ab^n a \) for \( n > 0 \)? ______
\( v_0 \) is the start node. \( w \) is a terminal node. A node labeled with both \( v_0 \) and \( w \) is both a start and terminal node.

Which finite state automaton correctly recognizes only words of the language \( ab^n \) for \( n > 0 \)?

Which finite state automaton correctly recognizes only words of the language \( a^n b \) for \( n > 0 \)?

Which finite state automaton correctly recognizes only words of the language \( ab^n a \) for \( n > 0 \)?

Which finite state automaton correctly recognizes only words of the language \( a^n b^n a \) for \( n > 0 \)?

The "quicker" power function version to calculate \( x^n \) was defined in the book and one of the homeworks as:

\[
\begin{align*}
\text{if } n \text{ is equal to 0, return 1.} \\
\text{if } n \text{ is even, compute result } = \text{power}(x, n/2) \text{ and return (result } \times \text{ result).} \\
\text{if } n \text{ is odd, return (} x \times \text{power}(x, n-1)).
\end{align*}
\]

Write the recurrence relation for the number of multiplies in this power function as

\[
C(n) = \begin{cases} 
\text{ } & \text{if } n \text{ is 0} \\
\text{ } & \text{if } n \text{ is even} \\
\text{ } & \text{if } n \text{ is odd}
\end{cases}
\]

What big-Oh complexity class is this quicker power function?
Extra Credit

Match the person to what the person is famous for. (1/2 point each)

____ Populatized outputting the string "hello, world" as a traditional first program to learn a language.
____ Invented the Merge sort algorithm.
____ Developed Unix along with Kernighan & Ritchie, created grep, co-designed the Go programming language at Google.
____ Known as the father of the C++ programming language.
____ Invented the Quicksort algorithm.
____ Number of transistors that can be placed inexpensively on an integrated circuit doubles approximately every two years.
____ Known as the father of the Linux operating system.
____ Known as the father of the Java programming language.
____ Described early single-memory, stored program architecture is now commonly known as the general purpose single-core computer.
____ Known as the father of the GNU Project and GCC.
____ Known as the father of the C programming language.
____ The speedup of a program using multiple processors in parallel computing is limited by the time needed for the sequential fraction of the program.
____ Known as the father of Lisp and garbage collection.
____ The Nobel prize of Computer Science named after this person.