By filling in the above and signing my name, I confirm I will complete this exam with the utmost integrity and in accordance with the Policy on Integrity of Scholarship.

Final
CSE 21
Spring 2012

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SubTotal ___________ (141 points = 100%)

Page 11 ___________ (10 points) Extra Credit (7%)

Total ___________

This exam is to be taken by yourself with closed books, closed notes, no electronic devices. You are allowed both sides of an 8.5"x11" sheet of paper handwritten by you.
Consider the following recurrence relation:

\[
G(n) = \begin{cases} 
1 & \text{if } n = 0 \\
G(n - 1) + 2n - 1 & \text{if } n > 0 
\end{cases}
\]

Calculate the first 6 terms for this recurrent relation. Then to the right calculate the sequence of differences between these terms. You may not need all the slots on the right.

<table>
<thead>
<tr>
<th>n</th>
<th>Sequences of differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>________________________</td>
</tr>
<tr>
<td>1</td>
<td>________________________</td>
</tr>
<tr>
<td>2</td>
<td>________________________</td>
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<tr>
<td>3</td>
<td>________________________</td>
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<tr>
<td>4</td>
<td>________________________</td>
</tr>
<tr>
<td>5</td>
<td>________________________</td>
</tr>
</tbody>
</table>

Use the calculated terms and the sequence of differences to help determine the closed-form solution for \( G(n) \).

\[ f(n) = \text{________________________} \]

Verify this with a proof by induction. Prove \( G(n) = f(n) \) for all \( n \) ___________.

Proof (Induction on \( n \)):

\[ \text{Base case: } \text{If } n = , \text{ the recurrence relation says } G(\_) = , \text{ and the closed-form solution says } f(\_) = \text{________________________} = , \text{ so } G(\_) = f(\_)\]  

\[ \text{Inductive step: Suppose as inductive hypothesis that } G(\_) = \text{________________________} \text{ for some } k \]  

\[ \text{Using the recurrence relation, } G(k) = \text{________________________} , \text{ by 2}^{\text{nd}} \text{ part of RR} \]

\[ = \text{________________________} , \text{ by IHOP} \]

\[ = \text{________________________} \]

\[ = \text{________________________} \]

So, by induction, \( G(n) = f(n) \) for all __________ (as __________).
Give a recurrence relation that describes the sequence 3, 6, 12, 24, 48, 96, 192, …

\[
G(n) = \begin{cases} 
& \text{____________________} \quad \text{if } n = 1 \\
& \text{____________________} \quad \text{if } n > 1 
\end{cases}
\]

Which of the general types of decompositions of recursive algorithms discussed in class would be good to use for each of the following?

Build a fractal like Koch snowflake

Binary search and merge sort

Reverse a string by moving last element to front, recurse on rest
\[(sa)^R \rightarrow a(s)^R\]

Check if a string is a palindrome

Calculate the factorial of \(n\) as a recursive computer program taking a single formal parameter

Reverse chars in an array in place

You may want to draw Venn diagrams (on your scratch paper) to help you answer the following few questions.

Professor Polly Morphism has 25 students in her Computer Architecture class, 15 students in her Discrete Math class, and 20 students in her Compilers class.

Assuming that there are no students who are taking more than one class from her, how many students does she have? ______

Assuming there are 5 students who take both Computer Architecture and Discrete Math at the same time and 7 students who take both Discrete Math and Compilers at the same and 6 students who take both Computer Architecture and Compilers at the same time and no student takes all three classes at the same time, how many total students does she have? ______

how many students are only in her Compilers class but not in her other classes? ______

how many students are in her Computer Architecture class but not in her other classes? ______

and how many students are in her Discrete Math class but not in her other classes? ______

Now instead of no student taking all three classes, what if the above question was modified to now specify that 2 of the above students take all three classes at the same time. How many total students does she have? ______

How many different strings can be formed by rearranging the letters in SUCCESSLESSNESS, using all the letters?

How many different strings of length 17 can be formed from a set of 26 refrigerator magnets A-Z?
How many (nonempty) strings of at most length 4 can be formed from a 26-symbol alphabet?

How many ways are there to rearrange the letters in the word PALINDROMES?

How many strings of length 4 can be formed from a 17-symbol alphabet where no 2 adjacent symbols are the same?

What is the value of P(7,4)? Your answer should be an actual number for this one.

What is the value of C(5,2)? Your answer should be an actual number for this one.

How many UCSD student ids are possible where a student id is made up of a single lower-case alpha character (a-z) followed by 8 digits (0-9)?

How many student ids have all different digits (no duplicates)? [Don't forget the leading char in the following.]

How many student ids contain only odd digits in the digits part of their ids?

How many student ids have at least one even digit?

How many student ids start with an odd digit and end with an even digit in the digits part of their ids?

How many student ids can be formed with at least one duplicate digit (for example, a004568912, d588683124, and y339979712)?

C(42,5) is the same value as C(42, ____)
C(38,1) is the same value as C(38, ____)
C(29,29) is the same value as C(29, ____)
Snow White has 50 one-dollar bills, which she wishes to divide up among the seven different dwarves. Each dwarf may receive any (integral) number of bills, from 0 to 50. How many different ways can she distribute this money?

An urn contains 12 balls numbered 0-11. Seven balls are drawn from the urn in sequence, and the numbers on the balls are recorded. How many ways are there to do this, if

all seven balls are drawn at once (one handful of seven balls)?

each ball is replaced before the next one is drawn?

when each ball is drawn it is not replaced?

With a 16-bit address register, how many different memory locations can be addressed? This is the same as asking how many different values (bit patterns) are there with a 16-bit binary string.

How many different possible eight character passwords are there made up of upper (A-Z) and lower (a-z) alpha characters and digits (0-9) and ten punctuation characters?

If there are 735 incoming students who need to take a particular lab class with 8 different sections of this lab class, there will always be a group of at least how many students who will try to enroll in any one section?

How many different ways are there to distribute 20 identical bones among 4 different dogs? (No dog can get a negative number of bones.)

Given the binary tree to the right

Specify the output for the following traversals

Preorder traversal: ____________________________

Inorder traversal: ____________________________

Postorder traversal: __________________________

Breadth-first traversal: ______________________

In Craps, a Field Bet is a bet that the next roll will be a 2, 3, 4, 9, 10, 11, or 12. It pays even money (1-to-1) for 3, 4, 9, 10, and 11, and pays double (2-to-1) for 2 and 12.

What is the probability of rolling a 2 or 12 (assume two fair 6-sided dice)?

\[ P(X= 2 \text{ or } 12) = \] ___________  
Non-reduced fractions are preferred.

What is the probability of rolling a 3, 4, 9, 10, or 11 (assume two fair 6-sided dice)?

\[ P(X= 3, 4, 9, 10, \text{ or } 11) = \] ___________  
Non-reduced fractions are preferred.

For a $1 bet on the Field, the payout for rolling a 2 or 12 is 2-to-1 (for example, $1 bet pays $2 + the original $1 bet for a total of $3) and the payout for rolling a 3, 4, 9, 10, or 11 is 1-to-1 (for example, $1 bet pays $1 + the original $1 bet for a total of $2), the Expected Value of the amount of money you will pull off the table in terms of \( P(X=x) \) is

\[ E(X) = 3 \times P(X = 2 \text{ or } 12) + 2 \times P(X = 3, 4, 9, 10, \text{ or } 11) + 0 \times P(X != 2, 3, 4, 9, 10, 11, \text{ or } 12) \]

Now replace the \( P(X=x) \) values with their numeric probabilities keeping your answer in terms of fractions vs. decimals. Non-reduced fractions are preferred.

\[ E(X) = 3 \times \frac{1}{36} + 2 \times \frac{1}{6} + 0 \times \frac{24}{36} = \] ___________

If your bet is $1 (costs you $1 to play), what is your expected return each time you make this kind of bet? Express your answer as a positive or negative non-reduced fraction.

\[ E(X) - 1 = \] ___________

A simple Pass Line bet Expected Value is approximately -1/70 or about -1.41%. Which is a better overall bet for you who is placing the bet - the Pass Line or the Field?

____________________

Big-Oh provides an(a) ____________ bound on the growth rate of a function while big-Omega provides an(a) ____________ bound on the growth rate of a function.

Given \( K_1 f(n) \leq g(n) \leq K_2 f(n) \), we say that "\( g \) is big-__________ of \( f \)" or "\( g \) is order \( f \).

\( K_1 f(n) \) represents the big-__________ of \( f \) while \( K_2 f(n) \) represents the big-__________ of \( f \).

With respect to the graph to the right,
the function labeled _____ represents big-Omega of \( f \) and
the function labeled _____ represents big-Oh of \( f \).
How many four-digit binary strings are there that do not contain 101 or 100? First draw the entire decision tree with 0s to the left and 1s to the right. Just one 0 or 1 per line.

Now cross out leaf nodes of any string that contains 101 or 100 in its path. The remaining leaf nodes should be paths that do not contain 101 or 100. How many such four-digit binary strings do not contain 101 or 100? _____

Match the big-Oh complexity class value to the graph. Write the letter associated with the function to the right that describes the big-Oh runtime complexity for that function.

- O(2^n) - B
- O(log_2 n) - D
- O(n) - A
- O(n^3) - C
- O(1) - E

Between which two plot lines would we find
- O(n log_2 n) - between ____ and ____
- O(n^3) - between ____ and ____

What is the name of the complexity class for each labeled function? (Not big-Oh – the English word name for the complexity class.)

A) __________________________
B) __________________________
C) __________________________
D) __________________________
E) __________________________
Match the algorithms with their recurrence relations and their run time complexities. Use the letters and numbers from the boxes to the right.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Recurrence Relation</th>
<th>Time Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Merge Sort</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Towers of Hanoi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bubble Sort</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Selection Sort</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Binary Tree Traversal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Binary Search</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sequential Search</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fibonacci sequence</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(recursive)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Array access</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fibonacci sequence</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(iterative as discussed in class and homework)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1) \( T(n) = T(n-1) + O(n) \)
2) \( T(n) = 2 T(n/2) + O(n) \)
3) \( T(n) = T(n/2) + O(1) \)
4) \( T(n) = T(n-1) + T(n-2) + O(1) \)
5) \( T(n) = T(n-1) + O(1) \)
6) \( T(n) = 2 T(n-1) + O(1) \)
7) \( T(n) = 2 T(n/2) + O(1) \)
8) \( T(n) = 2 T(n-1) + O(n) \)

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</tr>
</tbody>
</table>

A) \( O(2^n) \)
B) \( O(1) \)
C) \( O(n \log_2 n) \)
D) \( O(\log_2 n) \)
E) \( O(n!) \)
F) \( O(n^2) \)
G) \( O(n) \)

Construct a minimum spanning tree from the following network. Use the grayed network on the right to construct your msp. Hint: 9 vertices so msp should have 8 edges.

```
What is the total weight of the minimum spanning tree? ______
Is there more than one minimum spanning tree in this graph (yes or no)? ______

______ is the collection of all problems whose solutions can be checked (but not necessarily solved) in polynomial time.

______ is the collection of all problems that can be solved with an algorithm whose complexity is, at most, polynomial.
Given the initial order of ints in an array as: 3, 9, 6, 1, 4, 0, 5, 7 what is the order of the elements after 4 iterations of the selection sort algorithm covered in class and one of the HW exercises?

____  ____  ____  ____  ____  ____  ____  ____

Fill in the blanks to complete the iterative Fibonacci algorithm (similar to one of your HW assignments).

```c
long long fib_Iterative(int n)
{
    long long i_minus_2 = 0; // fib(0) = 0
    long long i_minus_1 = 1; // fib(1) = 1
    long long fib_i;
    if ( n <= 0 )            // n == 0
        return _____________________;
    if ( n == 1 )            // n == 1
        return _____________________;
    int i = 2;               // start at fib(2)
    while ( i <= n )
    {
        fib_i = ____________________ + ____________________; // fib(i) for this value of i
        i_minus_2 = ____________________; // set up i_minus_2 for next iteration
        i_minus_1 = ____________________; // set up i_minus_1 for next iteration
        ++i;
    }
    return ____________________;
}
```

Consider the following algorithm:

```c
for (i = 1; i <= 6; ++i)
{
    beep;
    for (j = 1; j <= 4; ++j)
    {
        beep;
    }
    for (k = 1; k <= 3; ++k)
    {
        for (l = 1; l <= 5; ++l)
        {
            beep;
        }
        for (m = 1; m <= 4; ++m)
        {
            beep;
        }
    }
}
```

How many times does the \texttt{beep} statement get executed? __________

Find a big-Theta estimate for the number of ways to choose a set of five or fewer elements from a set of size $n$. ______________
Give a postcondition for the following alg. that completely describes how the final value of \( i \) is related to \( x \).

**Precondition:** \( x \) is a positive odd integer.

\[
i ← 0
\]
\[
\text{while } i < x \text{ do}
\]
\[
i ← i + 2
\]

**Postcondition:** ______________________________________

The "quicker" power function version to calculate \( x^n \) was defined in one of the homeworks as:

- if \( n \) is equal to 0, return 1.
- if \( n \) is even, compute result = power( \( x \), \( n/2 \) ) and return (result * result).
- if \( n \) is odd, return (\( x \) * power( \( x \), \( n-1 \) )).

Write the recurrence relation for the number of multiplies in this power function as

\[
C(n) = \begin{cases} 
\text{__________________________} & \text{if } n \text{ is } 0 \\
\text{__________________________} & \text{if } n \text{ is even} \\
\text{__________________________} & \text{if } n \text{ is odd}
\end{cases}
\]

What big-Oh complexity class is this quicker power function? ________________

If you have an algorithm that when you double the number of elements it …

- doubles the number of comparisons, this algorithm is most likely in what big-Oh complexity class? ________
- squares the number of comparisons, this algorithm is most likely in what big-Oh complexity class? ________
- does not change the number of comparisons, this alg. is most likely in what big-Oh complexity class? ________
- increases the number of comparisons by 1, this alg. is most likely in what big-Oh complexity class? ________

A regular expression cannot be written to correctly recognize only valid strings that are palindromes or only valid strings in the form \( a^n b^n \) because regular expressions cannot ________________.

In grep (and vim), what metacharacter is used to specify/match the beginning of the line? ________

In grep (and vim), what metacharacter is used to match 1 or more of the preceding element? ________

In grep (and vim), what metacharacter is used to match 0 or more of the preceding element? ________
S is the start symbol. $a$ and $b$ are a terminal symbols.

1) $S \rightarrow aSa$
2) $S \rightarrow SbS$
3) $S \rightarrow Sb$
4) $S \rightarrow Sab$
5) $S \rightarrow aSb$
6) $S \rightarrow abS$
7) None of the above

Which context-free grammar correctly recognizes only words of the language $ab^n$ for $n \geq 0$? _____
Which context-free grammar correctly recognizes only words of the language $ab^n a$ for $n \geq 1$? _____
Which context-free grammar correctly recognizes only words of the language $a^n b^n$ for $n \geq 1$? _____

$v_0$ is the start node. $w$ is a terminal node. A node labeled with both $v_0$ and $w$ is both a start and terminal node.

A) ![Diagram A]
B) ![Diagram B]
C) ![Diagram C]
D) None of the above

Which finite state automaton correctly recognizes only words of the language $(ab)^n$ for $n \geq 0$? _____
Which finite state automaton correctly recognizes only words of the language $a(ba)^n$ for $n \geq 0$? _____
Which finite state automaton correctly recognizes only words of the language $a^n b^n a$ for $n \geq 1$? _____

Rick's rapper name is _______________________________________________.
Extra Credit

Match the person to what the person is famous for. (1/2 point each)

_____ Coined the term Unix.
_____ Known as the father of the GNU Project and GCC.
_____ Known as the father of the Java programming language.
_____ Invented Quicksort algorithm.
_____ Known as the father of Lisp and garbage collection.
_____ Known as the father of the C programming language.
_____ Known as the father of the analysis of algorithms.
_____ Co-founded Intel and described trend in number of transistors.
_____ First Turing Award winner.
_____ Known as the father of the Linux operating system.
_____ Known as an accomplished juggler believe it or not.
_____ Known as the father of the Fortran programming language.
_____ Has a well-known conference celebrating women in computing named after.
_____ The 'A' of AWK and extended the regular expression capabilities of the original grep with egrep.
_____ Formula to predict the theoretical maximum speed-up using multiple processors.
_____ Invented the single-source shortest path algorithm and the semaphore used in operating systems.
_____ Developed Unix along with Kernighan & Ritchie, co-designed the Go programming language at Google.
_____ Invented Merge sort algorithm and described early single-memory, stored program architecture we now commonly know as the general purpose computer.
_____ Known as the father of the C++ programming language.
_____ Visualized a nanosecond with a piece of wire just under a foot in length as the distance light travels in a nanosecond.