### Final
CSE 21
Fall 2010

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<tr>
<th>Page</th>
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Calculate the first 6 terms for the sum of the first $n$ natural numbers ($n = 1, n = 2, n = 3, \ldots, n = 6$). Then to the right calculate the sequence of differences between these terms. The first natural number is 1 (not 0). You may not need all the slots on the right.

$$\begin{array}{cc}
  n & \text{Sequences of differences} \\
  1 & \underline{\text{_______________________}} = \underline{\text{____}} \\
  2 & \underline{\text{_______________________}} = \underline{\text{____}} \quad \underline{\text{____}} \\
  3 & \underline{\text{_______________________}} = \underline{\text{____}} \quad \underline{\text{____}} \quad \underline{\text{____}} \\
  4 & \underline{\text{_______________________}} = \underline{\text{____}} \quad \underline{\text{____}} \quad \underline{\text{____}} \quad \underline{\text{____}} \\
  5 & \underline{\text{_______________________}} = \underline{\text{____}} \quad \underline{\text{____}} \quad \underline{____} \\
  6 & \underline{\text{_______________________}} = \underline{____} \\
\end{array}$$

Write the recurrence relation for the sum of the first $n$ natural numbers $T(n)$.

$$T(n) = \begin{cases} 
  \underline{\text{_______________________}} & \text{if } n \underline{______} \\
  \underline{\text{_______________________}} & \text{if } n \underline{______} 
\end{cases}$$

What is for the closed-form solution to the recurrence relation above?

$$f(n) = \underline{\text{_______________________}}$$

Verify this with a proof by induction. Prove $T(n) = f(n)$ for all $n \underline{______}$.

Proof (Induction on $n$):

$\underline{_______}$: If $n = \underline{____}$, the recurrence relation says $T(\underline{____}) = \underline{____}$, and the closed-form solution says $f(\underline{____}) = \underline{____} = \underline{____}$, so $T(\underline{____}) = f(\underline{____})$.

$\underline{_______}$: Suppose as inductive hypothesis that $T(\underline{____}) = \underline{____} \underline{____}$

for some $k \underline{______}$.

$\underline{_______}$: Using the recurrence relation, $T(k) = \underline{____} \underline{____}$, by $2^{nd}$ part of RR

$$= \underline{____} \underline{____}, \text{ by IHOP}$$

$$= \underline{____}$$

$$= \underline{____}$$

So, by induction, $T(n) = \underline{____} \underline{____}$ for all $\underline{____}$ (as $\underline{____}$).
Which is true about proof by induction on $k$ (IHOP on $k$ and IS on $k+1$) vs proof by induction on $k-1$ (IHOP on $k-1$ and IS on $k$)? ______

A) Different proofs (not same)  C) first (on $k$) uses weak induction and second (on $k-1$) uses strong induction
B) Both the same  D) first (on $k$) uses strong induction and second (on $k-1$) uses weak induction

You may want to draw Venn diagrams (on your scratch paper) to help you answer the following few questions.

Professor R.E. Cursion has 27 students in her Data Structures class, 14 students in her Discrete Math class, and 18 students in her Compilers class.

Assuming that there are no students who are taking more than one class from her, how many students does she have? ______

Assuming there are 4 students who take both Data Structures and Discrete Math at the same time and 4 students who take both Discrete Math and Compilers at the same and 5 students who take both Data Structures and Compilers at the same time and no student takes all three classes at the same time, how many students does she have? ______

What kind of problem is the previous question? __________________________________________________

What if the above question was modified to include 3 students who take all three classes at the same time. How many students does she have? ______

Mike has 5 different bikes and 4 different cars. He plans to ride a bike to and from work, and then take one of his cars to go out at night. How many different ways can he do this? ______

How many different strings can be formed by rearranging the letters in NONSENSELESSNESS, using all the letters?

_________________________________________

How many different strings of length 13 can be formed from a set of 26 refrigerator magnets A-Z?

_________________________________________

How many strings of length 5 can be formed from a 15-symbol alphabet where no 2 adjacent symbols are the same?

_________________________________________

What is the value of $P(6,2)$? Your answer should be an actual number for this one.

_________________________________________

What is the value of $C(7,3)$? Your answer should be an actual number for this one.

_________________________________________
How many 7-digit phone numbers are possible within an area code? Phone numbers can contain all zeros thru all nines.

How many 7-digit phone numbers have all different digits (no duplicates)?

How many 7-digit phone numbers contain only odd digits?

How many 7-digit phone numbers have at least one even digit?

How many 7-digit phone numbers start with an odd digit and end with an even digit?

How many 7-digit phone numbers can be formed with at least one duplicate digit (for example, 0045689, 5886831, and 3399797)?

The easiest way to solve the previous question is to use the technique called

How many (nonempty) strings of at most length 5 can be formed from a 26-symbol alphabet?

How many ways are there to rearrange the letters in the word TRONLEGACY?

Match which expression evaluates to the same value?

\[ \begin{array}{c|c}
   \text{expression} & \text{values} \\
   \hline
   \text{C}(26,6) & A) \text{C}(26,5) \\
   & B) \text{C}(26,0) \\
   \text{C}(26,26) & C) \text{C}(26,25) \\
   & D) \text{C}(26,20) \\
   \text{C}(26,1) & E) \text{C}(26,13) \\
\end{array} \]

If 1245 babies are born in December, there will always be a group of at least how many who were born on the same day of the month?

What kind of problem is the previous question?
An urn contains 10 balls numbered 0-9. Six balls are drawn from the urn in sequence, and the numbers on the balls are recorded. How many ways are there to do this, if

when each ball is drawn it is not replaced?

all six balls are drawn at once (one handful of six balls)?

each ball is replaced before the next one is drawn?

How many solutions (using only non-negative integers) are there to the following equation?

\[ x_0 + x_1 + x_2 + x_3 + x_4 + x_5 = 25 \]

How many different ways are there to distribute 16 identical bones among 6 different dogs? (No dog can get a negative number of bones.)

How many different ways are there to rearrange the 12-bit binary string (bit pattern) 10011000100?

What is the probability of rolling either a 6 or 8 (sum of two fair 6-sided dice will be 6 or 8)?

What is the probability of rolling a 4, 5, 6, 7, 8, 9, 10, or 11 (sum of two fair 6-sided dice will be a 4 - 11)?

Given the binary tree to the right

Specify the output for the following traversals

Preorder traversal: ________________________________

Inorder traversal: ________________________________

Postorder traversal: ________________________________
There are 38 possible outcomes in American roulette (1-36 and 0 and 00). The numbers 1-36 alternate between red and black while 0 and 00 are green.

What is the probability of a roulette ball landing in a red slot?

\[
P(X = \text{red}) = \underline{}\]

The payout for a bet on red is 1-to-1 (for example, $1 bet pays $1 + the original $1 bet for a total of $2), the Expected Value of the amount of money you will win in terms of \( P(X=x) \) is

\[
E(X) = 2 \cdot P(X = \text{red}) + 0 \cdot P(X \neq \text{red})
\]

Now replace the \( P(X=x) \) values with their numeric probabilities keeping your answer in terms of fractions vs. decimals. Reduced fractions are preferred.

\[
E(X) = 2 \cdot \underline{} + 0 \cdot \underline{} = \underline{}
\]

If your bet is $1 (costs you $1 to play), what is your expected return each time you make this kind of bet? Express your answer as a positive or negative reduced fraction.

\[
E(X) - 1 = \underline{}
\]

If you walk up to an American roulette table and see there have been 15 consecutive red winners before you got there, and you decide to place a bet on black, what is the probability of the roulette ball landing in a black slot for this bet?

\[
\underline{}
\]

Big-Oh provides a(n) ________ bound on the growth rate of a function while big-Omega provides a(n) ________ bound on the growth rate of a function.

Given \( K_1 f(n) \leq g(n) \leq K_2 f(n) \), we say that "\( g \) is big-__________ of \( f \)" or "\( g \) is order \( f \)."

\( K_1 f(n) \) represents the big-__________ of \( f \) while \( K_2 f(n) \) represents the big-__________ of \( f \).

With respect to the graph to the right, the function labeled _____ represents big-Oh of \( f \) and the function labeled _____ represents big-Omega of \( f \).
How many four-digit binary strings are there that do not contain 100 or 011? First draw a decision tree.

How many such four-digit binary strings that do not contain 100 or 011? _____

Match the time complexity class names in the box to the right with their big-Oh equivalent.

<table>
<thead>
<tr>
<th></th>
<th>O( 1 )</th>
<th>O( log_2 n )</th>
<th>O( n! )</th>
<th>O( 2^n )</th>
<th>O( n )</th>
<th>O( n^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A) polynomial</td>
<td>B) exponential</td>
<td>F) logarithmic</td>
<td>C) factorial</td>
<td>G) n log_2 n</td>
<td>D) constant</td>
<td></td>
</tr>
</tbody>
</table>

Match the algorithms with their recurrence relations and their run time complexities. Use the letters and numbers from the boxes to the right.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Recurrence Relation</th>
<th>Time Complexity</th>
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</thead>
<tbody>
<tr>
<td>Selection Sort</td>
<td></td>
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<tr>
<td>Binary Tree Traversal</td>
<td></td>
<td></td>
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<tr>
<td>Binary Search</td>
<td></td>
<td></td>
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<tr>
<td>Sequential Search</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fibonacci sequence</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(recursive)</td>
<td></td>
<td></td>
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<tr>
<td>Merge Sort</td>
<td></td>
<td></td>
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<tr>
<td>Towers of Hanoi</td>
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<tr>
<td>Bubble Sort</td>
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<tr>
<td>Fibonacci sequence</td>
<td></td>
<td></td>
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<tr>
<td>(iterative as shown in class)</td>
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<td></td>
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<tr>
<td>Array access</td>
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</tbody>
</table>

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<th>Algorithm</th>
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<th>Time Complexity</th>
</tr>
</thead>
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<tr>
<td>A) T(n) = T(n/2) + O(1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B) T(n) = T(n-1) + T(n-2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C) T(n) = T(n-1) + O(1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D) T(n) = 2 T(n-1) + O(1)</td>
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<td></td>
</tr>
<tr>
<td>E) T(n) = 2 T(n/2) + O(1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F) T(n) = T(n-1) + O(n)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G) T(n) = 2 T(n/2) + O(n)</td>
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<th>A) polynomial</th>
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<th>C) factorial</th>
<th>G) n log_2 n</th>
</tr>
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<tbody>
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<td>1) O( log_2 n)</td>
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<tr>
<td>2) O( n! )</td>
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<tr>
<td>3) O( n^2 )</td>
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<tr>
<td>4) O( n )</td>
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<tr>
<td>5) O( 2^n )</td>
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<tr>
<td>6) O( 1 )</td>
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<tr>
<td>7) O( n log_2 n)</td>
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</table>
Construct a minimum spanning tree from the following network. Use the grayed network on the right to construct your msp. Hint: 9 vertices so msp should have 8 edges.

What is the total weight of the minimum spanning tree? _______

Is there more than one minimum spanning tree in this graph (yes or no)? ______

In a collection of 69 coins, 1 coin is counterfeit and weighs 0.1 ounce less than the genuine coins. The genuine coins all weigh a known amount (say, 1.0 ounce). Find a good lower bound on the number of balance scale weighings needed to identify the fake coin.

_____________________

Instead of a balance scale in the above question, you had a sensitive bathroom scale (a scale that you can weigh only one set of objects at a time). Find a good lower bound on the number of time you need to use this device to identify the fake coin out of 69 coins.

_____________________

_____ is the collection of all problems whose solutions can be checked (but not necessarily solved) in polynomial time.

_____ is the collection of all problems that can be solved with an algorithm whose complexity is, at most, polynomial.

_____________________ is the collection of problems such that a polynomial-time solution to one of these problems would yield a polynomial-time solution to any problem in NP.

Given the initial order of ints in an array as: 8, 6, 0, 9, 3, 7, 5, 4 what is the order of the elements after 3 iterations of the selection sort algorithm covered in class and one of the HW exercises?

_____ _____ _____ _____ _____ _____ _____

A regular expression cannot be written to correctly recognize only valid strings that are palindromes or only valid strings in the form $a^n b^n$ because regular expressions cannot ________________.

In grep (and vim), what metacharacter is used to specify the end of line? ________

In grep (and vim), what metacharacter is used to match 0 or 1 of the preceding element? ________
$S$ is the start symbol. $a$ and $b$ are a terminal symbols.

1) $S \rightarrow Sab$
   $S \rightarrow ab$

2) $S \rightarrow aSa \mid bSb$
   $S \rightarrow a \mid b \mid \varepsilon$

3) None of the above

Which context-free grammar correctly recognizes only words of the language $a(ba)^n$ for $n \geq 0$? _____

Which context-free grammar correctly recognizes only words of the language $a^n b$ for $n \geq 0$? _____

Which two context-free grammars correctly recognizes only words of the language $(ab)^n$ for $n \geq 1$? _____  _____

$v_0$ is the start node. $w$ is a terminal node. A node labeled with both $v_0$ and $w$ is both a start and terminal node.

A)

B)

C)

D) None of the above

Which finite state automaton correctly recognizes only words of the language $a^n b^n a$ for $n \geq 1$? _____

Which finite state automaton correctly recognizes only words of the language $(ab)^n$ for $n \geq 1$? _____

Which finite state automaton correctly recognizes only words of the language $a^n b$ for $n \geq 1$? _____

Which of the following is another way of specifying $(ab)^n$ for $n \geq 1$ using regular expression metacharacters? _____

A) $(ab)^*$  B) $(ab)^+$  C) $(ab)^2$  D) $a^* b^*$  E) $a^+ b^+$  F) $[ab] \{n\}$  G) $a^2 b^2$
How many distinct pairs of array elements are there in an array of \( n \) elements?

Give your answer in terms of \( n \). ______________

Give the same answer as a combinatorics notation in terms of \( n \). ______________

Given the following recurrence relation, write the actual body of code for the function \( T(n) \). Just write the code for the base case and recursive case (do not worry about anything else).

\[
T(n) = \begin{cases} 
5 & \text{if } n = 0 \\
T(n-1) + n - 5 & \text{if } n > 0
\end{cases}
\]

```
long long int T( int n )
{
    // Assume n >= 0
    //
    ___________________________________________________________________
    ___________________________________________________________________
    ___________________________________________________________________
    ___________________________________________________________________
}
```
Extra Credit

Match the person to what the person is famous for. (1/2 point each)

_____ Wrote the first "hello world" program.
_____ Author of The Art of Computer Programming.
_____ Invented Quicksort algorithm.
_____ Invented the shunting-yard algorithm.
_____ Developed Unix along with Kernighan and Thompson.
_____ Known as the father of C.
_____ Known as the father of Pascal.
_____ One of the letters in BNF.
_____ Invented Merge sort algorithm.
_____ Influential in early regular expression work used in grep, awk, and most editors (not Brian Kernighan).
_____ Co-authored *Concrete Mathematics* (a blend of CONtinuous and disCRETE math) with Donald Knuth.
_____ Known as the father of Fortran.
_____ Has a well-known conference celebrating women in computing named after.
_____ The 'K' of AWK and the 'K' of K&R C.
_____ A theoretical device representing a computing machine to understand limits of computation named after.
_____ At one time his number was known as the largest number ever used in a serious mathematical proof.
_____ Outspoken critic of the goto statement; favored structured programming.
_____ Author of the TeX computer typesetting system and computer modern family of typefaces.
_____ Proved that what is now known as the halting problem is undecidable.
_____ Conceptualized the idea of machine-independent programming languages, which led to the development of COBOL

A) Alan Turing
B) C.A.R. Hoare
C) John von Neumann
D) Ron Graham
E) Dennis Ritchie
F) Edsger Dijkstra
G) Brian Kernighan
H) Grace Hopper
I) Niklaus Wirth
J) Donald Knuth
K) Ken Thompson
L) John Backus