On the Complexity of Graph Cuboidal Dual Problems for 3-D Floorplanning of Integrated Circuit Design

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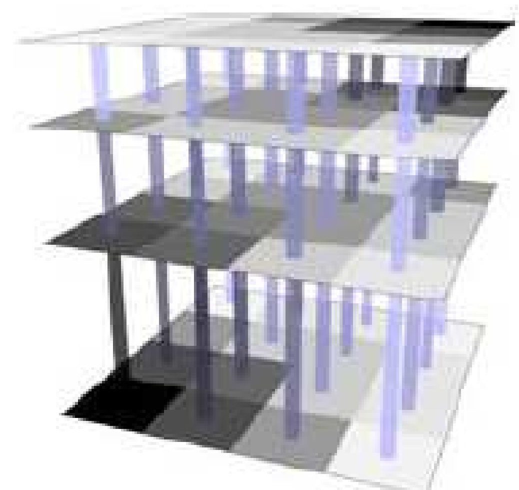
Goal of Today's Talk

- Discuss 2-D → 3-D
- Introduce a cuboidal dual problem which differentiates #dimensions
- Measurable complexity in the problems
- Hardness of 3-D cuboidal dual
- Hardness of 2.5-D cuboidal dual
  - Single layer, i.e. 2-D cuboidal dual
  - 3 or more layers
  - …

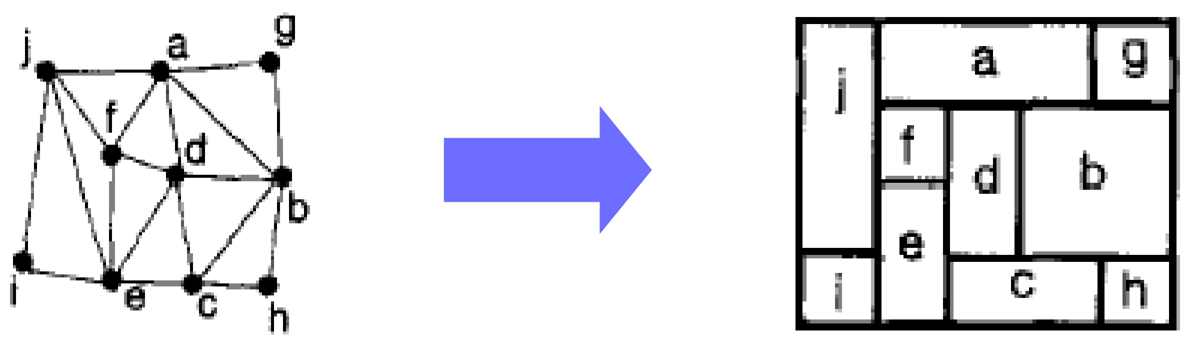
Introduction

- "Moore's law" enabled by
  - Reduction on lithographic structures
  - New technologies and methodologies
- 3-dimensional circuit
  - To overcome the interconnect bottleneck, ideally $\sqrt{n} \rightarrow \sqrt[3]{n}$
- 3-D challenges
  - Thermal behavior, cooling
  - Higher complexity in design, CAD, fabrication



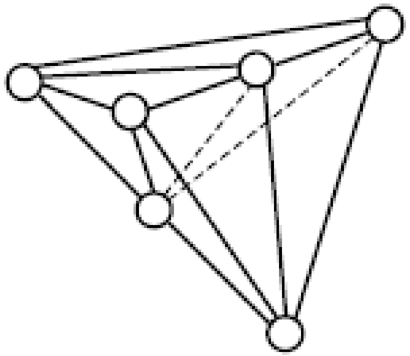
How much higher complexity?

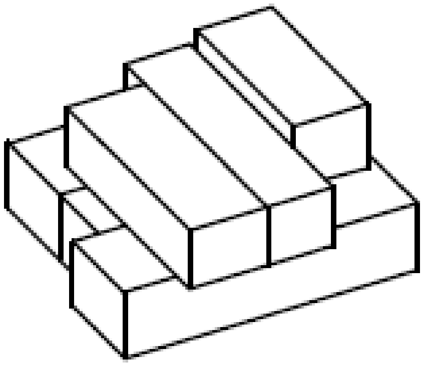
- Placement of sub-circuit blocks
- "Rectangular dual" formulation
  - Kozminski & Kinnen. "An algorithm for finding a rectangular dual of a planar graph for use in area planning for VLSI integrated circuits" *DAC*'84
- Planar graph *G* → Rectangle dissection with adjacency graph isomorphic to *G*



Graph Cuboidal Dual

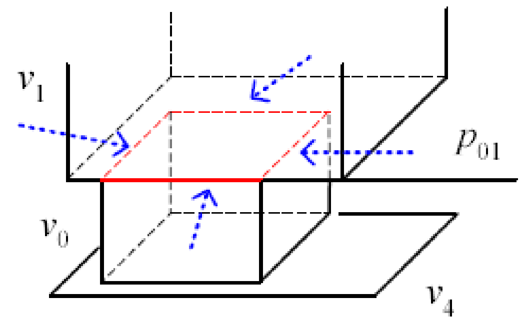
- Generalize to 3-D: Given a graph *G=(V,E)*, can we find a set of **cuboids** as *V* with **contact relations** as *E*?
  - No longer a “dissection” (for simplicity)
- Variations
  - 3-D
  - 2.5-D
    - Layered 3-D case
  - 2-D
    - Single layer 2.5-D case

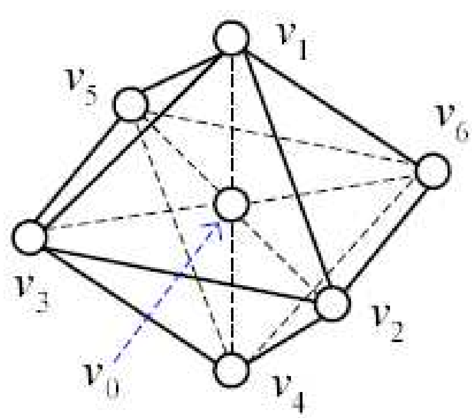




3-D Cuboidal Dual

- General 3-D cuboidal dual is NP-complete
- 3-COLOR reduces to 3-D cuboidal dual
  - Orientation of cuboids [xyz] → 3 colors
  - Gadget of directions

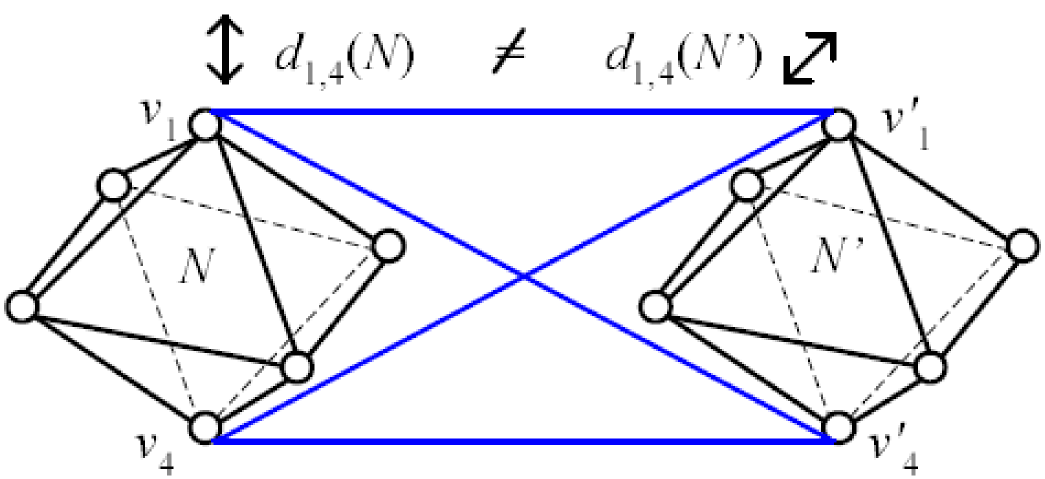


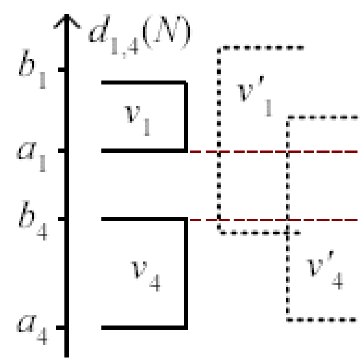


Lemma 1. In the cuboidal dual of the 7-vertex gadget, the cuboids of two opposite vertices on the octahedron (e.g. v_1, v_4) are on opposite sides of the central cuboid

3-D Cuboidal Dual (cont.)

- $d_{1,4}$ denotes the direction of $v_1 \rightarrow v_0 \rightarrow v_4$
  - 3 possible directions: x, y, z
- Enforcing 2 gadgets in different directions
  - Analogous to an edge in 3-COLOR

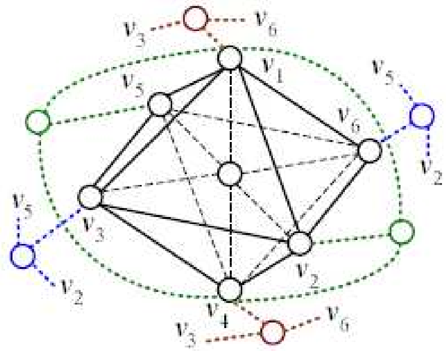


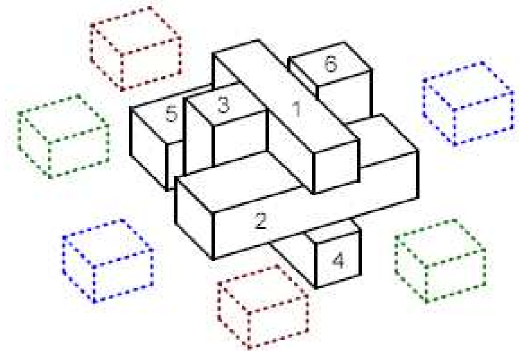


Bi-clique between $\{v_1, v_4\}$ and $\{v_1', v_4'\}$

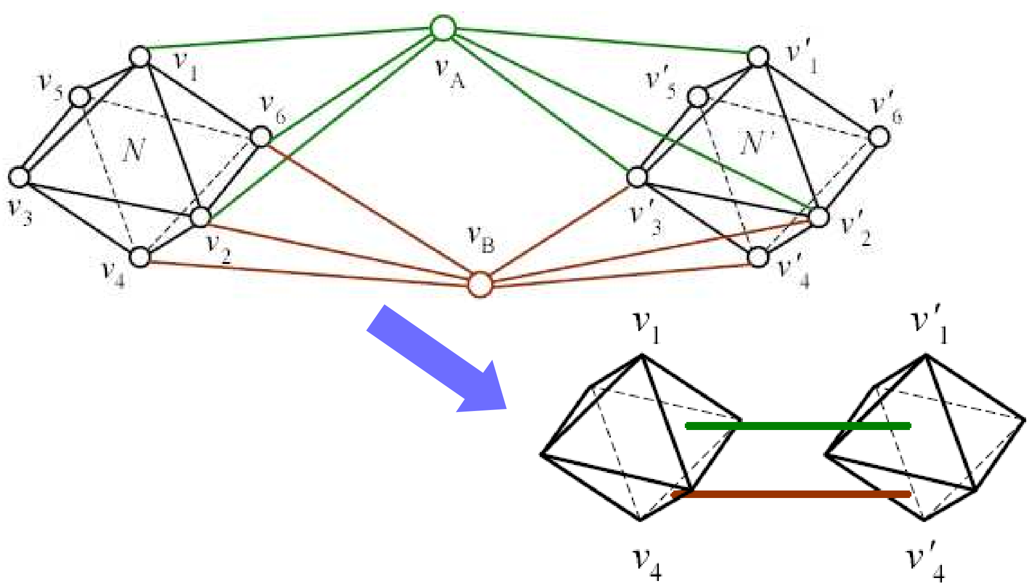
3-D Cuboidal Dual (cont.)

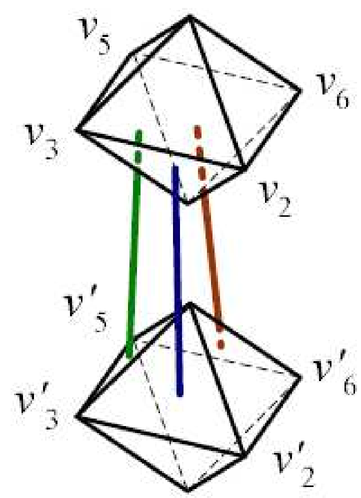
- Alignable gadgets
  - Add 6 vertices
  - Shape defined





- 2-alignment and 3-alignment

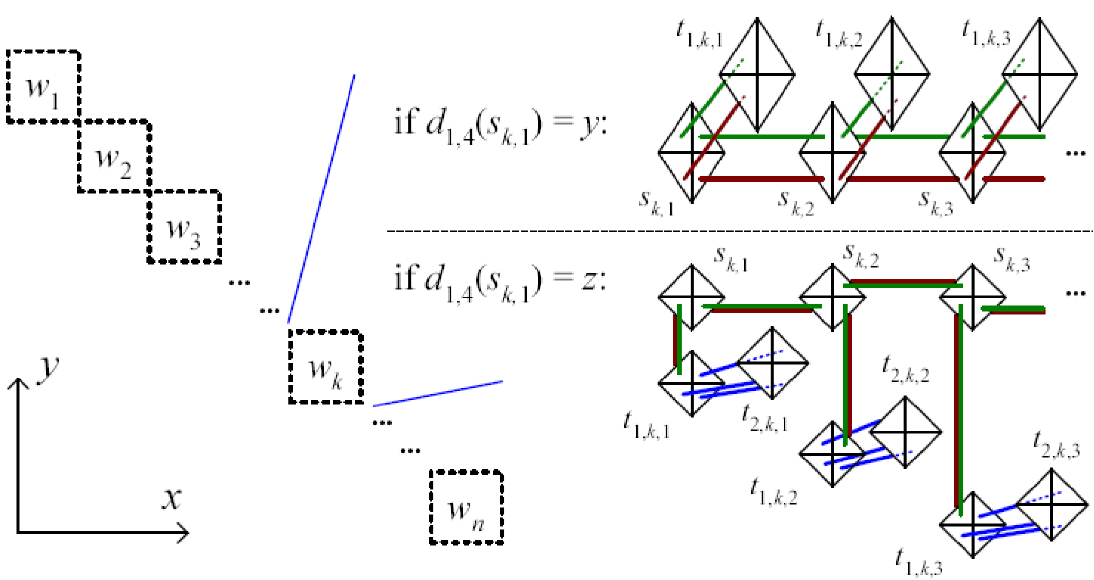




3-D Cuboidal Dual (cont.)

- *Theorem 1*. 3-COLOR reduces to 3-D cuboidal dual.
  - 3-COLOR graph $G_{3C} = (W, E_0) \rightarrow G = (V, E)$

Each w_i, 13-vertex gadgets $s_{i,1} \ldots s_{i,n}$

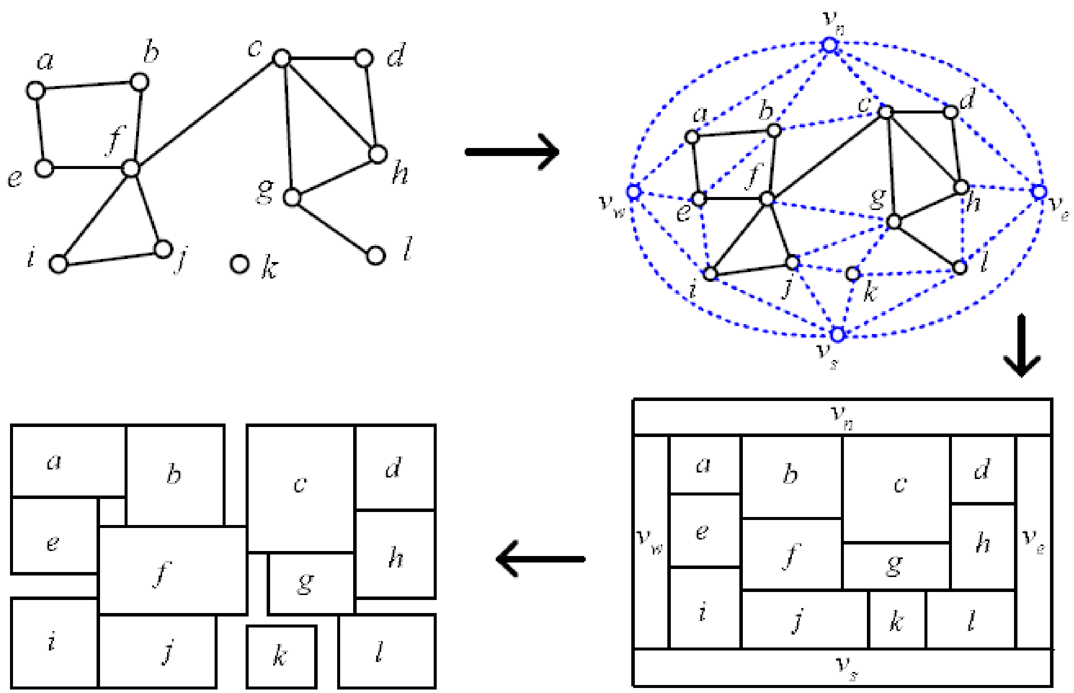


$s_{i,j}$ 2-aligns with $t_{1,i,j}$
$t_{1,i,j}$ 3-aligns with $t_{2,i,j}$
$t_{2,i,j}$ 2-aligns with $t_{3,i,j}$
$t_{3,i,j}$ 2-aligns with $u_{i,j}$

Each edge (w_i, w_j), enforce $u_{i,j}$ and $u_{j,i}$ in different directions (biclique connection)

2-D Cuboidal Dual

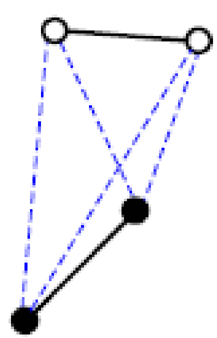
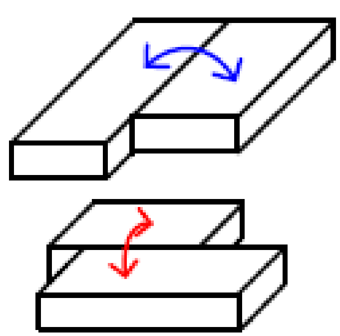
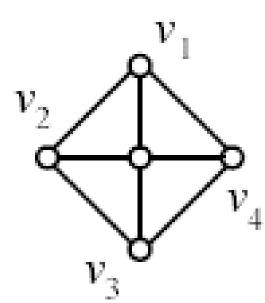
- *Theorem 2*. G has a 2-D cuboidal dual ←→ G can be drawn as a plane graph with no 3-vertex cycle with interior vertices

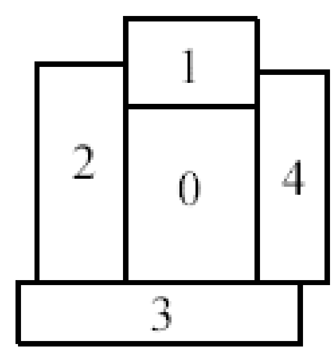


Conclusion of [Kozminski & Kinnen 84]

2.5-D Cuboidal Dual

- Given a layered graph
  - $G = (V, E, n, L : V \rightarrow \{1, \ldots, n\})$
  - (v_i, v_j) only exists when $|L(v_i) - L(v_j)| = 1$
  - Reduced freedom
- 2.5-D gadgets
  - Orthogonal contacts & diamond gadget

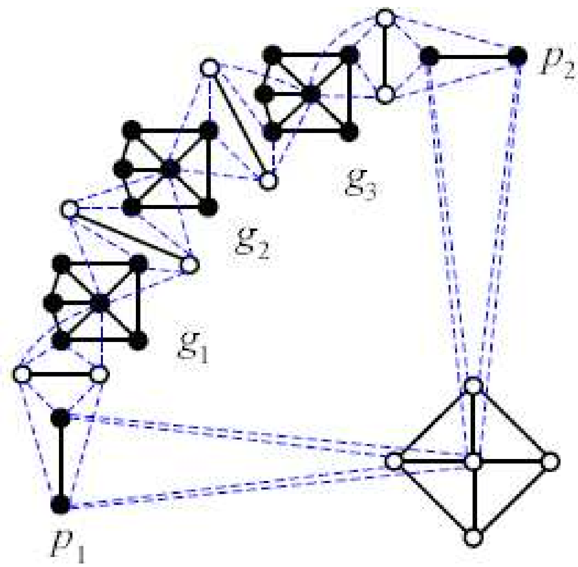




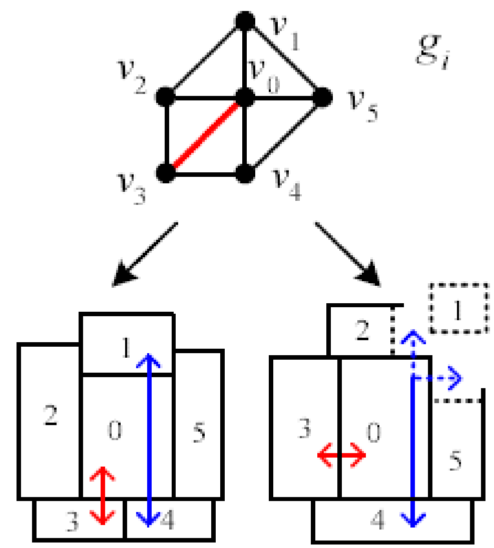


2.5-D Cuboidal Dual (cont.)

- *Theorem 3*. Planar 3-SAT reduces to 2.5-D cuboidal dual with 3 layers.

Clause gadget

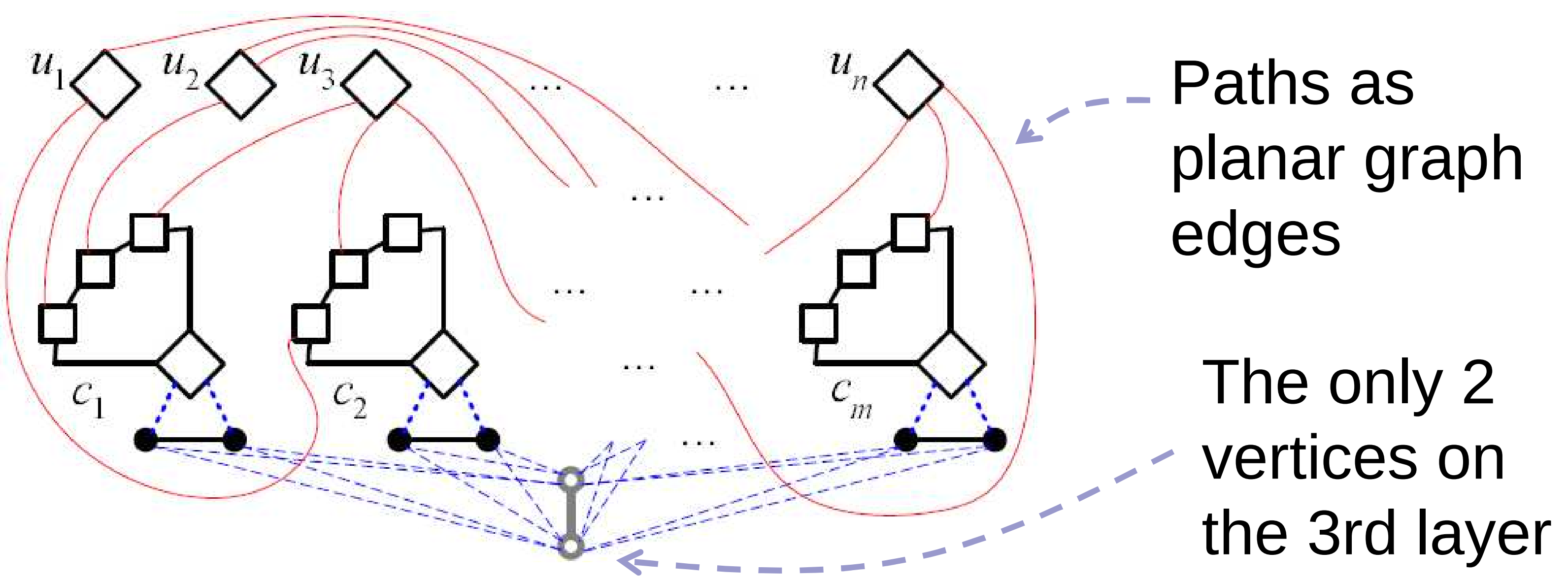




The 2-layer subgraph has a 2.5-D cuboidal dual $\leftarrow\rightarrow$ at least one 6-vertex gadget has horizontal $v_0 \rightarrow v_3$.

2.5-D Cuboidal Dual (cont.)

- Planar 3-SAT: *n* variables and *m* clauses
  - Each variable a diamond gadget
  - Each clause a clause gadget
  - *m* clause gadgets aligned by 3rd layer vertices



Hardness of Problem Variations

- 3-D cuboidal dual: NP-complete
- 2.5-D cuboidal dual
  - Single layer, i.e. 2-D: P
  - Double layer: *open*
  - 3 or more layers: NP-complete
- Analogy between colorability and cuboidal dual problems
  - 2-COLOR → 3-COLOR
  - 2 dimensions → 3 dimensions

Q & A

- Thank you for your attention