To Do

- Assignment 2 due Feb 19
  - Should already be well on way.
  - Contact us for difficulties etc.
- This lecture is a “bonus”: more advanced topic that is closer to the research frontier

Advanced Computer Graphics
CSE 190 [Winter 2016], Lecture 10
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Subdivision

- Very hot topic in computer graphics
- Brief survey lecture, quickly discuss ideas
- Detailed study quite sophisticated
  - See some of materials from readings

Advantages
- Simple (only need subdivision rule)
- Local (only look at nearby vertices)
- Arbitrary topology (since only local)
- No seams (unlike joining spline patches)

Outline

- Basic Subdivision Schemes
- Analysis of Continuity
- Exact and Efficient Evaluation (Stam 98)

Video: Geri’s Game (outside link)

Subdivision Surfaces

- Coarse mesh & subdivision rule
  - Smooth surface = limit of sequence of refinements
**Key Questions**

- How to refine mesh?
- Where to place new vertices?
  - Provable properties about limit surface

**Loop Subdivision Scheme**

- How to refine mesh?
  - Refine each triangle into 4 triangles by splitting each edge and connecting new vertices

**Loop Subdivision Scheme**

- Where to place new vertices?
  - Choose locations for new vertices as weighted average of original vertices in local neighborhood

**Loop Subdivision Scheme**

- Rules for extraordinary vertices and boundaries:

**Loop Subdivision Scheme**

Choose $\beta$ by analyzing continuity of limit surface

- Original Loop
  
  $\beta = \frac{3}{16} \left( \frac{5}{8} - \left( \frac{1}{8} + \frac{3}{16} \cos \frac{2\pi}{n} \right)^2 \right)$

- Warren
  
  $\beta = \begin{cases} 
  \frac{3}{16} & n > 3 \\
  \frac{3}{10} & n = 3 
  \end{cases}$

**Butterfly Subdivision**

- Interpolating subdivision: larger neighborhood
### Modified Butterfly Subdivision

Need special weights near extraordinary vertices

- For \( n = 3 \), weights are \( \frac{5}{12}, -\frac{1}{12}, -\frac{1}{12} \)
- For \( n = 4 \), weights are \( 0, -1/8, 0 \)
- For \( n \geq 5 \), weights are

\[
\frac{1}{n} \left( 1 + \cos \frac{2\pi}{n} + \frac{1}{2} \cos \frac{4\pi}{n} \right) j = 0...n - 1
\]

- Weight of extraordinary vertex = \( 1 - \sum \text{other weights} \)

### A Variety of Subdivision Schemes

- Triangles vs. Quads
- Interpolating vs. approximating

### More Exotic Methods

- Kobbelt’s subdivision:

### More Exotic Methods

- Number of faces *triples* per iteration: gives finer control over polygon count

### Subdivision Schemes

[Zorin & Schröder]
Outline

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Analyzing Subdivision Schemes

- Limit surface has provable smoothness properties

Analyzing Subdivision Schemes

- Start with curves: 4-point interpolating scheme

Subdivision Matrix

- How are vertices in neighborhood refined? (with vertex renumbering like in last slide)

\[
\begin{pmatrix}
  v^{(0)}_1 \\
  v^{(0)}_2 \\
  v^{(0)}_3 \\
  v^{(0)}_4 \\
  v^{(0)}_5 \\
  v^{(0)}_6 \\
  v^{(0)}_7 \\
  v^{(0)}_8
\end{pmatrix} =
\begin{pmatrix}
  0 & 1 & 0 & 0 & 0 \\
  \frac{9}{16} & \frac{9}{16} & \frac{9}{16} & \frac{9}{16} & 0 \\
  0 & 0 & 1 & 0 & 0 \\
  0 & \frac{9}{16} & \frac{9}{16} & \frac{9}{16} & \frac{9}{16} \\
  0 & 0 & 0 & 1 & 0 \\
  \frac{v_0}{16} & \frac{v_1}{16} & \frac{v_2}{16} & \frac{v_3}{16} & \frac{v_4}{16}
\end{pmatrix}
\]

\[
\begin{pmatrix}
  v^{(1)}_1 \\
  v^{(1)}_2 \\
  v^{(1)}_3 \\
  v^{(1)}_4 \\
  v^{(1)}_5 \\
  v^{(1)}_6 \\
  v^{(1)}_7 \\
  v^{(1)}_8
\end{pmatrix}
\]

So, 5 new points depend on 5 old points

Subdivision Matrix

- How are vertices in neighborhood refined? (with vertex renumbering like in last slide)

\[
V^{(n+1)} = S V^{(n)}
\]

After \( n \) rounds:

\[
V^{(n)} = S^n V^{(0)}
\]
### Convergence Criterion

Expand in eigenvectors of $S$

\[
\lambda V^{(n)} = S V^{(0)}
\]

$S = \sum_{i} \lambda_i e_i$

$V^{(0)} = \sum_{i} a_i e_i$

$V^{(n)} = \sum_{i} a_i \lambda_i^n e_i$

**Criterion I: $|\lambda| < 1$**

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### Translation Invariance

- For any translation $t$, want:

\[
\begin{pmatrix}
V_2^{(n)} + t \\
V_2^{(n+1)} + t \\
V_3^{(n)} + t \\
V_4^{(n+1)} + t \\
V_5^{(n)} + t
\end{pmatrix} = S
\begin{pmatrix}
V_2^{(n)} + t \\
V_2^{(n+1)} + t \\
V_3^{(n)} + t \\
V_4^{(n+1)} + t \\
V_5^{(n)} + t
\end{pmatrix}
\]

$s1 = 1$

**Criterion III: $e_0 = 1$, all other $|\lambda| < 1$**

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### Smoothness Criterion

- Case 1: $|\lambda_1| > |\lambda_2|$
  - Points can be anywhere in space spanned by $e_1, e_2$
  - No longer have smoothness guarantee

**Criterion IV: Smooth iff $\lambda_0 = 1 > |\lambda_1| > |\lambda|$.**

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### Continuity and Smoothness

- So, what about 4-point scheme?
  - Eigenvalues $= 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}, \frac{1}{8}$
  - $e_0 = 1$
  - Stable ✓
  - Translation invariant ✓
  - Smooth ✓
2-Point Scheme

- In contrast, consider 2-point interpolating scheme
  \[ \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \\ 0 & 0 \end{pmatrix} \]
  - Support = 3
  - Subdivision matrix

Continuity of 2-Point Scheme

- Eigenvalues = 1, 1/2, 1/2
- \( e_0 = 1 \)
- Stable
- Translation invariant
- Smooth:
  - Not smooth; in fact, this is piecewise linear

For Surfaces...

- Similar analysis: determine support, construct subdivision matrix, find eigenstuff
  - Caveat 1: separate analysis for each vertex valence
  - Caveat 2: consider more than 1 subdominant eigenvalue
  - Ref’s smoothness condition: \( \lambda_0 \rightarrow 1 > |\lambda_1| > |\lambda_2| > |\lambda_i| \)
  - Points lie in subspace spanned by \( e_1 \) and \( e_2 \)
    - If \(|\lambda_i| > |\lambda_j|\), neighborhood stretched when subdivided, but remains 2-manifold

Fun with Subdivision Methods

Behavior of surfaces depends on eigenvalues

- Real
- Complex
- Degenerate

(recall that symmetric matrices have real eigenvalues)

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Practical Evaluation

- Problems with Uniform Subdivision
  - Exponential growth of control mesh
  - Need several subdivisions before error is small
  - Ok if you are “drawing and forgetting”, otherwise …
- (Exact) Evaluation at arbitrary points
  - Tangent and other derivative evaluation needed
- Paper by Jos Stam SIGGRAPH 98 efficient method
  - Exact evaluation (essentially take out “subdivision”)
  - Smoothness analysis methods used to evaluate
Isolated Extraordinary Points

- After 2+ subdivisions, isolated “extraordinary” points where irregular valence
- Regular region is usually easy
  - For example, Catmull Clark can treat as B-Splines

Subdivision Matrix

\[
C_1 = AC_0,
\]
\[
C_n = AC_{n-1} = A^nC_0
\]

Eigen Space

\[
C_1 = AC_0,
\]
\[
C_n = AC_{n-1} = A^nC_0 \quad A = \begin{pmatrix} S & 0 \\ S_{11} & S_{12} \end{pmatrix}
\]
\[
AV = VA \quad A = VA \quad V^{-1}
\]
\[
\bar{C}_n = \bar{A}A^{n-1}C_0 = \bar{A}A^{n-1}V^{-1}C_0
\]
\[
s_{k,n}(n,v) = C_0^T A^{n-1} (P_iAV)^T b(n,v)
\]
\[
C_0 = V^{-1}C_0
\]

Comments

- Computing Eigen-Vectors is tricky
  - See Jos’ paper for details
  - He includes solutions for valence up to 500
- All eigenvalues are (abs) less than one
  - Except for lead value which is exactly one
  - Well defined limit behavior
- Exact evaluation allows “pushing to limit surface”
Curvature Plots

See Stam 98 for details

Summary

- Advantages:
  - Simple method for describing complex, smooth surfaces
  - Relatively easy to implement
  - Arbitrary topology
  - Local support
  - Guaranteed continuity
  - Multiresolution

- Difficulties:
  - Intuitive specification
  - Parameterization
  - Intersections

[Pixar]