There are 2 questions for a total of 10 points.

(5) 1. Assume you have functions $f$ and $g$ such that $f(n)$ is $O(g(n))$. For each of the following statements, decide whether it is true or false and give a proof or counterexample.
   - $\log_2 f(n)$ is $O(\log_2(g(n)))$
   - $f(n^2)$ is $O(g(n^2))$

(5) 2. The Fibonacci numbers $F_0, F_1, F_2, \ldots$ are defined by the rule
   
   \[
   F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2}
   \]
   
   Show by induction that $F_n = \frac{1}{\sqrt{5}} \left[ \left( \frac{1+\sqrt{5}}{2} \right)^n - \left( \frac{1-\sqrt{5}}{2} \right)^n \right]$. 